Some iffy comments on Khoo 2022

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[1] Handout at: http://www.thonygillies.org/wp-content/uploads/khoo-amc-central-2023.pdf

1 Overview

THIS BOOK IS AMBITIOUS.[2] I'm going to talk about some of the details because the details are where the dragons live.[3]

[2] To keep things snappy: *MoI* = *The meaning of 'if'*.

[3] Phrase shamelessly stolen from Kai von Fintel.

2 Core semantics

Definition 1. A conditional $A \to C$ is true relative to:

1. context c;
2. sequence σ;
3. modal base m (eventually decomposed into modal base h and ordering source g relative to time t, world w); and
4. admissible partition $\mathbb{Z}$

iff (roughly) the nearest A-world is a C-world, nearness being a function of the sequence, modal base, and partition.

Comment 1. The meaning of *if*, in other words, is relative to no less than seven (7!) parameters, which must be some sort of record.

3 Partition dependence

In one segment of *MoI* the truth of a conditional $A \to C$ depends on a contextually relevant partition.[4] Very trendy.[5]

[4] Of logical space? Of the modal base? Of the common ground? I'm not entirely sure: the theory says the first, but some of the examples in *MoI* treat it as one of the latter two.

[5] I mean, look at this rogues' gallery that all employ this gambit: Egan 2016; von Fintel & Gillies 2010; Moss 2015; Roberts 2020; Yalcin 2007.

Definition 2. A partition $\mathbb{Z}$ is <u>admissible</u> for a conditional $A \to C$ at σ only if:

1. for every $Z \in \mathbb{Z}$: $Z \cap m^{\mathbf{A}}_{\sigma} \neq \emptyset$; and
2. for some $Z \in \mathbb{Z}$: $Z \nsubseteq \mathbf{C}$ and $Z \nsubseteq \bar{\mathbf{C}}$.

Example. There has been a murder. The murderer acted alone, and there are only two suspects (the butler, the gardener). We are trying to solve the case and so the QUD is: Who did it? $\square$

Consider:

(1) If the butler didn't do it, the gardener did. $\neg B \rightarrow G$

Observation 1. The QUD-induced partition $\{\mathbf{B}, \mathbf{G}\} = \{\mathbf{B}, \bar{\mathbf{B}}\}$ is ruled out as inadmissible for the conditional (1).[6]

[6] Note also that $\{\mathbf{B}, \mathbf{G}\} = \{\bar{\mathbf{G}}, \mathbf{G}\}$.

Comment 2. If this is an inadmissible partition for (1) then I fear that the definition is not latching onto a natural target, or the connection between QUD and partition is not what we thought.

4 Is this really a nearness analysis?

On the one hand, *if*s are Stalnaker & Thomason 1970 conditionals: they get a truth value at a sequence[7] and the core semantics validates expected characteristic entailments (conditional excluded middle, strong centering).[8]

[7] A.k.a. a (family of) well-founded ordering that is reflexive, anti-symmetric, and connected.

[8] For the record: I am not a fan of either.

On the other hand, they're not.

Definition 3. The refined content $\Uparrow\mathbf{S}$ determined by a state $\mathbf{S}$ is defined as:

$$\Uparrow\mathbf{S} = \{\sigma\colon\ \forall w \in \mathbf{S}\ \&\ \forall v \notin \mathbf{S} |\ w >_\sigma v\}$$

Definition 4. A state (set of worlds) $\mathbf{S}$ supports $A \rightarrow C$ iff $\Uparrow\mathbf{S} \subseteq \mathsf{A} \rightarrow \mathsf{C}$.

Observation 2. If $m_w = \mathbf{S}$ for each $w \in \mathbf{S}$ then $\mathbf{S}$ supports $A \rightarrow C$ iff $(\mathbf{S} \cap \mathbf{A}) \subseteq \mathbf{C}$.

Part of me is sympathetic.[9] Then again ....

[9] Gillies 2004, 2020

Definition 5. A formula A is persistent iff for any $\mathbf{S}, \mathbf{S}'$: if $\mathbf{S}$ supports A then $\mathbf{S}'$ supports A whenever $\mathbf{S}' \subseteq \mathbf{S}$.

Observation 3. Conditionals are not persistent given the core semantics in *MoI*.

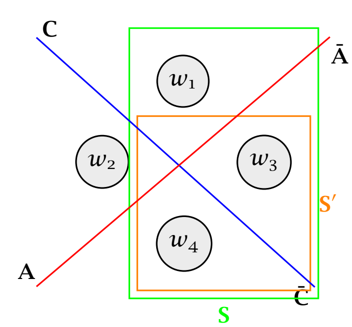

Figure 1: Non-persistence

Counterexample. Let $m_{w_i} = \mathbf{S}$ for $1 \leq i \leq 4$ where:

- $\mathbf{S} = \{w_1, w_3, w_4\}$;
- $\mathbf{A} = \{w_1, w_2\}$; and
- $\mathbf{C} = \{w_1, w_3\}$.

Taking refined contents is not monotonic: $\Uparrow \mathbf{S}' \not\subseteq \Uparrow \mathbf{S}$ even though $\mathbf{S}' \subseteq \mathbf{S}$. See Figure 1. □

Comment 3. From a state-based semantics point of view, you expect broadly $\Box/\forall$-like environments like conditionals to be persistent and their negations to be broadly $\Diamond/\exists$-like and not-persistent.

5 A type mismatch?

Let's say we want to add an existential modal like *maybe* to mix with our *if*s.

(2) a. Maybe if he didn't tell Tom he told Harry.
$\Diamond(\neg T \to H)$

b. If he didn't tell Tom, maybe he told Harry.
$\neg T \to \Diamond H$

Definition 6 (First try)**.** For any A, w, and m:

$$[\![\Diamond A]\!]^{w,m} = 1 \text{ iff } \exists v \in m_w : \ [\![A]\!]^{v,m} = 1$$

This is exactly what you'd expect.[10]

[10] Mostly. I don't really know how the world parameter gets determined here. Maybe it is the first world in the sequence that is the index for any upstairs conditional?

Definition 7 (Second try)**.** For any atom p, and any A, σ, and m:[11]

[11] Treat worlds as valuations.

1. $[\![p]\!]^{\sigma,m} = 1$ iff $\sigma_w(p) = 1$
2. (boolean clauses you'd expect)
3. $[\![\Diamond A]\!]^{\sigma,m} = 1$ iff $\exists \sigma' : \ \sigma_w \in m_\sigma \ \& \ [\![A]\!]^{\sigma',m} = 1$

Nice.[12]

[12] Note that for descriptive A, there is a derivative sense of *truth-at-w*: being true at a sequence whose first member is w. This doesn't make sense for modals.

Similar thing for hedged conditionals and conditional hedges:

(3) a. Probably if he didn't tell Tom he told Harry.
$\trianglerighteq (\neg T \to H)$

b. If he didn't tell Tom, he probably told Harry.
$\neg T \to \trianglerighteq H$

Comment 4. The suspicion is that this pattern repeats. (And is why the treatment of, say, embedded conditionals and import/export has a whiff of non-compositionality.)

6 King of Bavaria and Veltman's sisters

Counterfactuals in *MoI* in *MoI*: an intriguing mix of a broadly interventionist approach (Pearl, 2009; Halpern, 2016) and a broadly restricted-modality approach (Kratzer, 1979, 2012).[13]

[13] This is meant to be continuous with the sequence semantics stuff that precedes it. I admit to not being completely sure about that and so will just begin with a clean slate here.

Definition 8. For a counterfactual $A \Box\!\!\rightarrow C$ at w (and time t):

1. Historical modal base:

$$h^{\mathbf{A}}_{w,t} = \left\{ \mathbf{P}\colon \begin{array}{l} \mathbf{P} = \mathbf{A} \text{ or} \\ \mathbf{P} \text{ is a pre-}t \text{ truth or} \\ \mathbf{P} \text{ is a post-}t \text{ sufficiency} \end{array} \right\}$$

2. Ordering:

$$g^{\mathbf{A}}_{w,t} = \{\mathbf{P}\colon \mathbf{P} \text{ is a post-}t \text{ truth } \underline{\text{independent}} \text{ of A}\}$$

Example. Three sisters — Ann, Billie, Carol — own just one bed, large enough for two of them but too small for all three.[14] Every night at least one of them sleeps on the floor. Whenever Ann sleeps in the bed and Billie sleeps in the bed, Carol sleeps on the floor. Tonight, Billie is sleeping in the bed, Ann is on the floor, and Carol is in bed. (See Figure 2.) □

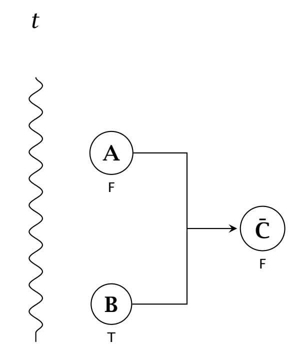

Figure 2: Veltman's sisters

[14] The example is from Veltman 2005 but has the shape as the "King of Bavaria" example in Kratzer 1989.

(4) If Ann were sleeping in bed, then Carol would be on the floor.

Observation 4. The analysis in *MoI* predicts this.[15]

[15] Officially (4) is "non-factual". It just seems false to me but YMMV.

Comment 5. Here the semantics in *MoI* disagrees with Kratzer 1989 and agrees with Veltman 2005.

7 Cautious monotonicity

How does the counterfactual in *MoI* relate to other, more familiar conditional logics? I have a somewhat surprising partial answer.

Example. Two switches, A and B, are connected to a light. If both switches are up, the light is on. How are the positions of A and B set? I'm glad you asked. By a combination of a coin toss, a die roll, and a drawing of a card from a standard deck.

1. Switch B is up if the coin comes up heads and the roll is even.
2. For switch A, if the coin comes up heads then there is a subsequent card draw. If the card is red then A is up.

As it happens, the coin came up heads, the roll was odd, and the card was red, and so the light is off. (See Figure 3.) $\square$

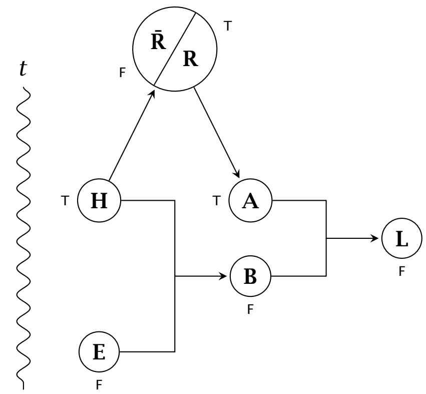

Figure 3: A dumb way to turn on a light

Now consider:

(5) a. If the roll had been even, switch B would have been up.
$E \mathrel{\Box\!\!\rightarrow} B$

b. If switch B had been up, the roll would have been even.
$B \mathrel{\Box\!\!\rightarrow} E$

Observation 5. $E \mathrel{\Box\!\!\rightarrow} B$ and $B \mathrel{\Box\!\!\rightarrow} E$.

Proof.
$E \mathrel{\Box\!\!\rightarrow} B$: $h_w^{\mathbf{E}} = \{\mathbf{E}, (\mathbf{E} \wedge \mathbf{H}) \equiv \mathbf{B}, \ldots\}$ and $g_w^{\mathbf{E}} = \{\mathbf{H}, \ldots\}$. These jointly entail **B**.

$B \mathrel{\Box\!\!\rightarrow} E$: $h_w^{\mathbf{B}} = \{\mathbf{B}, (\mathbf{E} \wedge \mathbf{H}) \equiv \mathbf{B}, \ldots\}$ entails **E**. $\square$

However, consider these counterfactuals:

(6) a. If the roll had been even, the light would have been on.
$E \mathrel{\Box\!\!\rightarrow} L$

b. If switch B had been up, the light would have been on.
$B \mathrel{\Box\!\!\rightarrow} L$

These differ only in their conditionally equivalent antecedents.

Observation 6. $E \mathrel{\Box\!\!\rightarrow} L$ but not $B \mathrel{\Box\!\!\rightarrow} L$.

Proof.
$E \mathrel{\Box\!\!\rightarrow} L$: **H** is independent of **E** and **R** is independent of **E** and so $h_w^{\mathbf{E}}$ and $g_w^{\mathbf{E}}$ entail **L**.

$B \mathrel{\Box\!\!\rightarrow} L$: **R** is not independent of **B** and **A** is not independent of **B**. More carefully: (i) **H** governs **B**; (ii) **H** governs $\mathbf{R} \vee \bar{\mathbf{R}}$ and so governs **R**; and **R** governs **A**. $\square$

Comment 6. This doesn't seem right empirically. Both (6a) and (6b) seem true.

Comment 7. Interesting!

Definition 9. A conditional $\Rightarrow$ satisfies (LCE) iff it satisfies

$$(A \Rightarrow C) \supset (B \Rightarrow C)$$

whenever $A \Rightarrow B$ and $B \Rightarrow A$.

Definition 10. The (flat) conditional logic $\mathbb{P}$ is the smallest logic containing all propositional tautologies plus:[16]

[16] This axiomatization first appears (I think) in Burgess 1981.

1. $A \Rightarrow A$ (Refl)
2. $(A \Rightarrow B \wedge A \Rightarrow C) \supset (A \Rightarrow (B \wedge C))$ (RDM)
3. $(A \Rightarrow B \wedge C) \supset (A \Rightarrow B)$ (RUM)
4. $((A \Rightarrow B) \wedge (A \Rightarrow C)) \supset ((A \wedge B) \Rightarrow C)$ (CM)
5. $((A \Rightarrow C) \wedge (B \Rightarrow C)) \supset ((A \vee B) \Rightarrow C)$ (Disj)

Observation 7. Any logic at least as strong as $\mathbb{P}$ satisfies (LCE).

Observation 8. The example above is a countermodel for (LCE) for the counterfactual in *MoI*.

Corollary 1. The counterfactual in *MoI* is not at least as strong as the conditional in $\mathbb{P}$.

Comment 8. Each characterizing validity of $\mathbb{P}$ is involved in deriving (LCE). However, it is difficult to see just which ones because interventions are not defined for logically complex antecedents.

Definition 11 (Ordering*)**.** For any **A** and **B**:

$$g_{w,t}^{\mathbf{A} \wedge \mathbf{B}} = \{\mathbf{P} \colon \mathbf{P} \text{ is a post-}t \text{ truth } \underline{\text{independent}} \text{ of both A and B}\}$$

Observation 9. Assuming Ordering*, the same example above is a countermodel to (CM).

Proof. $E \mathrel{\Box\!\!\rightarrow} B$ and $E \mathrel{\Box\!\!\rightarrow} L$ are both true but $(E \wedge B) \mathrel{\Box\!\!\rightarrow} L$ is not true. $\square$

Comment 9. You might be tempted to revise Ordering* so that $g_{w,t}^{\mathbf{A} \wedge \mathbf{B}}$ includes **P** only if **P** is independent of <u>either</u> **A** or **B**. Don't give in to temptation.

8 Closing arguments

There is more stuff in the book and more things to say about that stuff but by now I think you can see the sorts of things I'd want to say.

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