Some iffy comments on Khoo 2022

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1 Overview

THIS BOOK IS AMBITIOUS.[1] It tries to bring together insights from philosophy, linguistics, and logic to shed light on the many persistent and stubborn mysteries about conditionals — what they mean, how they express those meanings, and how they fit in to how we exchange information about the world and its happenings even while we remain ignorant about a great many things.

[1] To keep things snappy: *MoI* = *The meaning of 'if'*.

There are a lot of books published in philosophy these days. If I'm honest, too many. But few are as ambitious in scope, method, and target as is this one and for that it deserves high praise. I want to be clear about that because what follows might not seem to be dripping with praise. To be fair though, that wouldn't make for a very useful or entertaining session.

I like to think of doing philosophy as not unlike being a lawyer.[2] So Justin's job in writing this (quite long!) book is to advocate for a (quite large!) bundle of ideas about how we think about and express iffy things. My job as critic is to act as opposing counsel: see whether some of those ideas will break under cross-examination and, when they do, to see what crimes I can get them to confess to.

[2] Likely more accurately: a TV lawyer.

Having said that, I don't know that anything I have to say is an objection as such. This is just me working through some things and seeing where they go.

Time, not to mention your attention, is a scarce resource and so I will focus on just a few key cogs in the various analyses in *MoI*. That's the good news. The bad news is that what I have to say will be about some nitty-gritty details in how those cogs can and can't turn. In my defense, that is as it should be for a book about the meaning of *if*: the details are where the dragons live.[3]

[3] Phrase shamelessly stolen from Kai von Fintel.

2 Core semantics

The core analysis is this:

Definition 1. A conditional $A \to C$ is true relative to:

1. context c;
2. sequence σ (equivalently: a linear order over the set of worlds);
3. modal base m (eventually decomposed, in effect, into modal base h and ordering source g relative to time t and world w); and
4. admissible partition $\mathbb{Z}$

iff (roughly) the nearest A-world is a C-world, nearness being a function of the sequence, modal base, and partition.

Comment 1. The meaning of *if*, in other words, is relative to no less than seven (7!) parameters, which must be some sort of record. To be fair, and mercifully, not all parts of the book invoke all of the parameters all of the time. Though that kind of makes the point that the various incarnations of the analysis are not as unified as you would like.

My plan is to raise some issues about how these parameters fit together according to the theory in *MoI*.[4]

[4] Notation: $A, B, \ldots$ for sentences, $\mathbf{A}, \mathbf{B}, \ldots$ for corresponding sets of worlds, $\mathsf{A}, \mathsf{B}, \ldots$ for sets of sequences.

3 Partition dependence

In one segment of *MoI* the truth of a conditional $A \to C$ depends on a contextually relevant partition.[5] Different partition, possibly different truth values, and possibly different probabilities. Partitions are determined by the Question Under Discussion (QUD) in the context.

[5] Of logical space? Of the modal base? Of the common ground? I'm not entirely sure: the theory says the first, but some of the examples in *MoI* treat it as one of the latter two.

There's nothing objectionable in any of this. Partitions, partition-dependence, and QUDs are very trendy.[6]

[6] I mean, look at this rogues' gallery that all employ this gambit: Egan 2016; von Fintel & Gillies 2010; Moss 2015; Roberts 2020; Yalcin 2007.

But what might be objectionable are the constraints *MoI* places on admissible partitions. Context/QUD fixes a partition, subject to the partition meeting two side conditions.

Definition 2. A partition $\mathbb{Z}$ is admissible for a conditional $A \to C$ at σ only if:

1. for every $Z \in \mathbb{Z}$: $Z \cap m_{\sigma}^{\mathbf{A}} \neq \emptyset$; and
2. for some $Z \in \mathbb{Z}$: $Z \nsubseteq \mathbf{C}$ and $Z \nsubseteq \bar{\mathbf{C}}$.

Admissibility is curated to rule out certain partitions in some exotic examples. However perfectly ordinary contexts, with perfectly ordinary QUDs, give rise to inadmissible partitions.

Example. There has been a murder. The murderer acted alone, and there are only two suspects (the butler, the gardener). We are trying to solve the case and so the QUD is: Who did it? □

Consider:

(1) If the butler didn't do it, the gardener did. $\neg B \to G$

Observation 1. The QUD-induced partition $\{\mathbf{B}, \mathbf{G}\} = \{\mathbf{B}, \bar{\mathbf{B}}\}$ is ruled out as inadmissible for the conditional (1).[7]

[7] Note also that $\{\mathbf{B}, \mathbf{G}\} = \{\bar{\mathbf{G}}, \mathbf{G}\}$.

Proof. This partition flouts both constraints: (i) not every cell overlaps with $m_\sigma^{\bar{\mathbf{B}}}$; and (ii) there is no cell that entails neither $\mathbf{G}$ nor $\bar{\mathbf{G}}$ (because $\mathbf{B} = \bar{\mathbf{G}}$). □

Comment 2. If this counts as an inadmissible partition for (1) then I fear that the definition is not latching onto a natural target, or the connection between QUD and partition is not what we thought.

In what follows I will set partition-dependence aside.

4 Is this really a nearness analysis?

On the one hand, in *MoI* there is a lot emphasis on the idea that $A \to C$ is, fundamentally, a Stalnaker & Thomason 1970 type conditional: it gets a truth value at a sequence[8] and the core semantics validates the characteristic entailment patterns of conditional excluded middle and strong centering.[9]

[8] A.k.a. a (family of) well-founded ordering $\preceq_w$ that is reflexive, anti-symmetric, and connected.

[9] For the record: I am not a fan of either.

But, on the other hand, when it comes to a state supporting a conditional, the ordering information is washed away and what is important is that the state supports some strict conditional content.

Definition 3. The refined content $\Uparrow \mathbf{S}$ determined by a state $\mathbf{S}$ is defined as:

$$\Uparrow \mathbf{S} = \{\sigma : \forall w \in \mathbf{S} \;\&\; \forall v \notin \mathbf{S} |\; w >_\sigma v\}$$

Definition 4. A state (set of worlds) $\mathbf{S}$ supports $A \to C$ iff $\Uparrow \mathbf{S} \subseteq \mathsf{A} \to \mathsf{C}$.

Observation 2. If $m_w = \mathbf{S}$ for each $w \in \mathbf{S}$ then $\mathbf{S}$ supports $A \to C$ iff $(\mathbf{S} \cap \mathbf{A}) \subseteq \mathbf{C}$.

Proof. Suppose $\mathbf{S}$ supports $A \to C$ and consider any $v \in \mathbf{S} \cap \mathbf{A}$. Since $v \in \mathbf{S}$ there is some σ such that $\sigma_w = v$. Since $\mathbf{S}$ supports $A \to B$, it follows that $\sigma(\mathbf{A}) \in \mathbf{B}$ and so $v \in \mathbf{B}$. Now suppose $(\mathbf{S} \cap \mathbf{A}) \subseteq \mathbf{B}$ and let σ be any sequence in $\Uparrow \mathbf{S}$. Since $m_w = \mathbf{S}$ for every $w \in \mathbf{S}$, it follows that $\sigma(\mathbf{A}) \in \mathbf{S}$. Since $\sigma(\mathbf{A}) \in \mathbf{S} \cap \mathbf{A}$ it then follows that $\sigma(\mathbf{A}) \in \mathbf{B}$ and thus that $\mathbf{S}$ supports $A \to C$. $\square$

Of course maybe this is exactly what is intended. Part of me is sympathetic to that: I have defended various versions of broadly state-based semantics that treat *if* as a (dynamic) strict conditional.[10]

[10] Gillies 2004, 2020

But what *MoI* presents is not exactly a strict conditional story, even at the level of support-at-a-state. That is because the process of going from a state (set of worlds) to a refined state (set of sequences of worlds) introduces some unexpected results.

Definition 5. A formula A is persistent iff for any $\mathbf{S}, \mathbf{S}'$: if $\mathbf{S}$ supports A then $\mathbf{S}'$ supports A whenever $\mathbf{S}' \subseteq \mathbf{S}$.

More generally: persistence is preservation of support as ignorance shrinks.

Observation 3. Conditionals are not persistent given the core semantics in *MoI*.

Counterexample. Let $m_{w_i} = \mathbf{S}$ for $1 \leq i \leq 4$ where:

- $\mathbf{S} = \{w_1, w_3, w_4\}$;
- $\mathbf{A} = \{w_1, w_2\}$; and
- $\mathbf{C} = \{w_1, w_3\}$.

Note that $A \to C$ is supported by $\mathbf{S}$: the only $\mathbf{A}\bar{\mathbf{C}}$-world is w_2 and it is ranked last by every sequence in $\Uparrow \mathbf{S}$ since it is the only world not in $\mathbf{S}$. But — and this is the key thing — taking refined contents is not monotonic: $\Uparrow \mathbf{S}' \not\subseteq \Uparrow \mathbf{S}$ even though $\mathbf{S}' \subseteq \mathbf{S}$. As a result there are two sequences in $\Uparrow \mathbf{S}'$ in which $w_2 > w_1$ and two sequences in which $w_1 > w_2$. See Figure 1. $\square$

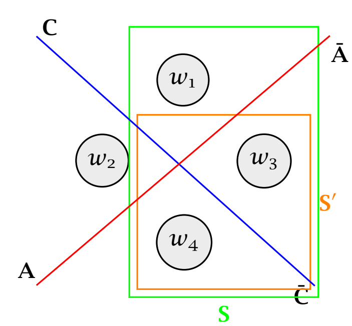

Figure 1: Non-persistence

Comment 3. This is surprising since from a state-based semantics point of view, you expect broadly $\square/\forall$-like environments like conditionals to be persistent and their negations to be broadly ◇/∃-like and therefore not-persistent. Here it is the opposite.

At this point I am not quite sure I fully get why *MoI* takes the detour through the Stalnaker & Thomason conditional only to wash out its main effects, especially when it appears that that detour introduces unexpected artifacts?

5 A type mismatch?

OK, confession time: who gets what it means for a conditional's truth value to be evaluated at a sequence and for its content to be a set of sequences (rather than, say, a set of worlds)? I want to put stress on this.

High altitude version of the worry: in general, for a happy compositional treatment of an index shifting operator capable of being embedded and embedding other things, you want the truth conditions to fundamentally be about goings on at indices. The core semantics in *MoI* does not do this: conditionals get truth values at sequences according to whether or not various worlds meet certain conditions. This is a red flag since worlds and sequences are different types of objects.

So, let's say we want to add an existential modal like *maybe* to mix with our *if*s.

(2) a. Maybe if he didn't tell Tom he told Harry.
$\Diamond(\neg T \to H)$

b. If he didn't tell Tom, maybe he told Harry.
$\neg T \to \Diamond H$

Note that *maybe* can occur either above or below *if*.

We might try, as a first pass, the gestured-at way of incorporating an existential modal into the core semantics from *MoI*:

Definition 6 (First try)**.** For any A, w, and m:

$$[\![\Diamond A]\!]^{w,m} = 1 \text{ iff } \exists v \in m_w\colon\ [\![A]\!]^{v,m} = 1$$

This is exactly what you'd expect: *maybe* shifts the point of evaluation, looking for a world in the modal base where its prejacent is true.[11] Terrific! Except, of course, if its prejacent is a conditional since those only get truth values at sequences.

[11] Mostly. I don't really know how the world parameter gets determined here. Maybe it is the first world in the sequence that is the index for any upstairs conditional?

Obvious solution: give a recursive definition of truth-at-a-sequence.

Definition 7 (Second try)**.** For any atom p, and any A, σ, and m:[12]

1. $[\![p]\!]^{\sigma,m} = 1$ iff $\sigma_w(p) = 1$
2. (boolean clauses you'd expect)
3. $[\![\Diamond A]\!]^{\sigma,m} = 1$ iff $\exists\sigma'$: $\sigma'_w \in m_\sigma$ & $[\![A]\!]^{\sigma',m} = 1$

In other words: *maybe* looks for a sequence whose first member is in the modal base and checks to see if its prejacent is true at the shifted sequence.[13] Terrific!

However. The core semantics in *MoI* assigns truth values at sequences but based on properties of worlds: $A \to C$ is true at σ iff its first world where A is true that is in m_σ is a world where C is true. But if $C = \Diamond B$, then we are again stuck because $\Diamond B$ is assigned truth values at sequences, not worlds.

Or take, for instance, hedged conditionals and conditional hedges:

(3) a. Probably if he didn't tell Tom he told Harry.
$\trianglerighteq(\neg T \to H)$

b. If he didn't tell Tom, he probably told Harry.
$\neg T \to \trianglerighteq H$

In (3a) *probably* takes wide scope over a conditional and so needs to latch onto the kind of probability in Chapter 5 of *MoI* suitable for sets of sequences. But in (3b) it takes narrow scope and so would seem to need to latch onto the more mundane probabilities of sets of worlds.

Comment 4. The suspicion is that this pattern repeats. (And is why the treatment of, say, embedded conditionals and import/export has a whiff of non-compositionality.)

6 King of Bavaria and Veltman's sisters

The approach to counterfactuals in *MoI* in *MoI* is an intriguing mix of a broadly interventionist approach (Pearl, 2009; Halpern, 2016) and a broadly restricted-modality approach (Kratzer, 1979, 2012).[14]

[12] Treat worlds as valuations; as in *MoI*, σ_w is the first world in σ.

[13] Note that for descriptive A, there is a derivative sense of *truth-at-w*: being true at a sequence whose first member is w. This doesn't make sense for modals.

[14] This is meant to be continuous with the sequence semantics stuff that precedes it. I admit to not being completely sure about that and so will just begin with a clean slate here.

Start with a sufficiency network that models the dependencies (deterministic and otherwise) or “laws”, and records which atomic facts are true and which are false. A counterfactual is, roughly, a restricted necessity operator over a domain. But the domain is built up from two distinct sets of premises.

Definition 8. For a counterfactual $A \,\Box\!\!\rightarrow C$ at w (and time t):

1. Historical modal base:

$$h^{\mathbf{A}}_{w,t} = \left\{ \mathbf{P} \colon \begin{array}{l} \mathbf{P} = \mathbf{A} \text{ or} \\ \mathbf{P} \text{ is a pre-}t \text{ truth or} \\ \mathbf{P} \text{ is a post-}t \text{ sufficiency} \end{array} \right\}$$

2. Ordering:

$$g^{\mathbf{A}}_{w,t} = \{\mathbf{P} \colon \mathbf{P} \text{ is a post-}t \text{ truth } \underline{\text{independent}} \text{ of A}\}$$

Clearly, the notion of independence is key.

To see this in action, consider an example.[15]

[15] The example is from Veltman 2005 but has the shape as the “King of Bavaria” example in Kratzer 1989.

Example. Three sisters — Ann, Billie, Carol — own just one bed, large enough for two of them but too small for all three. Every night at least one of them sleeps on the floor. Whenever Ann sleeps in the bed and Billie sleeps in the bed, Carol sleeps on the floor. Tonight, Billie is sleeping in the bed, Ann is on the floor, and Carol is in bed. (See Figure 2.) □

(4) If Ann were sleeping in bed, then Carol would be on the floor.

This counterfactual does not seem true. Why couldn’t Billie be on the floor if Ann had been in bed?

Observation 4. The analysis in *MoI* predicts this.[16]

[16] Officially (4) is “non-factual”. It just seems false to me but YMMV.

The reason is simple: neither **A** nor **B** are independent of $\bar{\mathbf{C}}$. So some worlds quantified over are **B**-worlds and some are $\bar{\mathbf{B}}$ worlds.

Comment 5. Here the semantics in *MoI* disagrees with Kratzer 1989 and agrees with Veltman 2005.

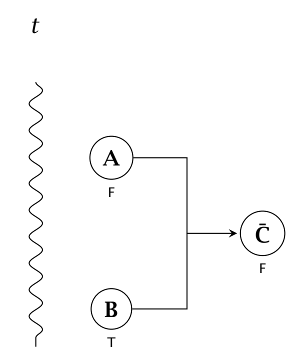

Figure 2: Veltman’s sisters

7 Cautious monotonicity

Famously, interventionist approaches are somewhat limited in scope: they are most at home for flat, unembedded conditional fragments. But where they overlap in coverage with full-throated logics of conditionals, there are interesting questions about exactly where they are located in logical space. The same question arises here for the analysis in *MoI*. I do not have a full answer, but have a somewhat surprising partial answer.

Example. Two switches, A and B, are connected to a light. If both switches are up, the light is on. How are the positions of A and B set? I'm glad you asked. By a combination of a coin toss, a die roll, and a random drawing of a card from a standard deck.

1. Switch B is up if the coin comes up heads and the roll is even.
2. For switch A, if the coin comes up heads then there is a subsequent card draw. If the card is red then A is up.

As it happens, the coin came up heads, the roll was odd, the card came up red, and so the light is off. (See Figure 3.) □

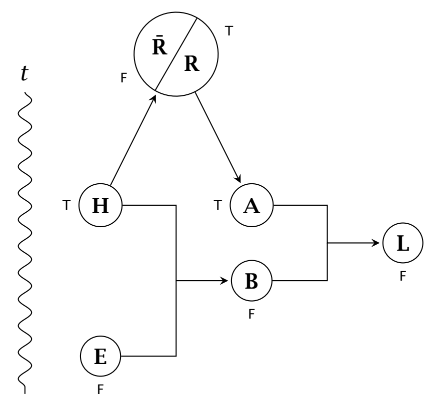

Figure 3: A dumb way to turn on a light

Now consider:

(5) a. If the roll had been even, switch B would have been up.
$E \boxright B$

b. If switch B had been up, the roll would have been even.
$B \boxright E$

These are both predicted to be true.

Observation 5. $E \boxright B$ and $B \boxright E$.

Proof.
$E \boxright B$: $h^{\mathbf{E}}_w = \{\mathbf{E}, (\mathbf{E} \wedge \mathbf{H}) \equiv \mathbf{B}, \ldots\}$ and $g^{\mathbf{E}}_w = \{\mathbf{H}, \ldots\}$. These two premise sets entail **B**.

$B \boxright E$: $h^{\mathbf{B}}_w = \{\mathbf{B}, (\mathbf{E} \wedge \mathbf{H}) \equiv \mathbf{B}, \ldots\}$ and this on its own entails **E**. □

However, consider these counterfactuals:

(6) a. If the roll had been even, the light would have been on.
$E \boxright L$

b. If switch B had been up, the light would have been on.
$B \boxright L$

The key is that these differ only in their conditionally equivalent antecedents. But they are not predicted to have the same truth values.

Observation 6. $E \boxright L$ but not $B \boxright L$.

Proof.
$E \boxright L$: $h_w^{\mathbf{E}} = \{\mathbf{E}, (\mathbf{E} \wedge \mathbf{H}) \equiv \mathbf{B}, \mathbf{R} \supset \mathbf{A}, (\mathbf{A} \wedge \mathbf{B}) \equiv \mathbf{L}\}$. **H** is independent of **E** and **R** is independent of **E**. Thus $h_w^{\mathbf{E}}$ and $g_w^{\mathbf{E}}$ entail **B** and entail **A**, and so entail **L**.

$B \boxright L$: the crucial fact is that **H** is <u>not</u> independent of **B** and so **R** is <u>not</u> independent of **B** and **A** is <u>not</u> independent of **B**.

More carefully: (i) **H** governs **B**; (ii) **H** governs $\mathbf{R} \vee \bar{\mathbf{R}}$ and so governs **R**; and **R** governs **A**. Therefore **B** and **R** share a root and **B** and **A** share a root. Thus **B** and **R** are connected and **B** and **A** are connected. Hence **B** and **R** are not independent and **B** and **A** are not independent. Therefore $\mathbf{R} \notin g_w^{\mathbf{B}}$ and $\mathbf{A} \notin g_w^{\mathbf{B}}$. □

Comment 6. This doesn't seem right empirically. Both (6a) and (6b) seem true.

Comment 7. Interesting!

Definition 9. A conditional ⇒ satisfies (LCE) iff it satisfies

$$(A \Rightarrow C) \supset (B \Rightarrow C)$$

whenever $A \Rightarrow B$ and $B \Rightarrow A$.

Observation 7. The example above is a countermodel for (LCE) for the counterfactual in *MoI*.

Definition 10. The (flat) conditional logic $\mathbb{P}$ is the smallest logic containing all propositional tautologies plus:[17]

1. $A \Rightarrow A$ (Refl)
2. $(A \Rightarrow B \wedge A \Rightarrow C) \supset (A \Rightarrow (B \wedge C))$ (RDM)
3. $(A \Rightarrow B \wedge C) \supset (A \Rightarrow B)$ (RUM)
4. $((A \Rightarrow B) \wedge (A \Rightarrow C)) \supset ((A \wedge B) \Rightarrow C)$ (CM)
5. $((A \Rightarrow C) \wedge (B \Rightarrow C)) \supset ((A \vee B) \Rightarrow C)$ (Disj)

[17] This axiomatization first appears (I think) in Burgess 1981.

Observation 8. Any logic at least as strong as $\mathbb{P}$ satisfies (LCE).

Proof. The proof invokes each of (Refl), (RDM), (RUM), (Disj), and — tellingly — (CM). □

Corollary 1. The counterfactual in *MoI* is not at least as strong as the conditional of $\mathbb{P}$.

Comment 8. Since each characterizing validity of $\mathbb{P}$ is involved in deriving (LCE), and we have a counterexample to (LCE) for the *MoI* counterfactual, at least one of those validities must fail for the counterfactual in *MoI*. However, it is difficult to see just which ones because the interventions are not defined for logically complex antecedents. So in particular it is hard to see whether the counterfactual in *MoI* validates (CM) and (Disj).

It is highly unobvious how to fix this for (Disj), but it is (more or less) straightforward how to patch things for (CM).[18]

[18] Forgive the abuse of notation here: since **A** and **B** are propositions, ∧ doesn't belong between them.

Definition 11 (Ordering*)**.** For any **A** and **B**:

$$g_{w,t}^{\mathbf{A}\wedge\mathbf{B}} = \{\mathbf{P} \colon \mathbf{P} \text{ is a post-}t \text{ truth } \underline{\text{independent}} \text{ of both } \mathbf{A} \text{ and } \mathbf{B}\}$$

Observation 9. Assuming Ordering*, the same example above is a countermodel to (CM).

Proof. $E \mathrel{\Box\!\!\rightarrow} B$ and $E \mathrel{\Box\!\!\rightarrow} L$ are both true but $(E \wedge B) \mathrel{\Box\!\!\rightarrow} L$ is not true. The reason is the same as before: **R** and **A** are not independent of **B** and so not independent of both **E** and **B**. □

Comment 9. You might be tempted to revise Ordering* so that $g_{w,t}^{\mathbf{A}\wedge\mathbf{B}}$ includes **P** only if **P** is independent of <u>either</u> **A** or **B**. Don't give in to temptation.

8 Closing arguments

I guess if I'm going to press the courtroom model, here is where I would sum up. I'm not going to do that, though. There is more stuff in the book and more things to say about that stuff but by now I think you can see the sorts of things I'd want to say.

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