Comments on "Assertability semantics"

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I LIKE THIS PAPER. Assertability semantics (AS) belongs to a family of approaches to understanding the meaning of ignorance-expressing language that I find natural and elegant and worth thinking a lot about.

You should read this: Hawke & Steinert-Threlkeld 2018.

1 Quick background sketch

From Veltman 1996; Gillies 2004.

Definition 1 (Update semantics w/indicatives). Key clauses for $[\cdot]\colon \mathbf{S} \to \mathbf{S}$:

1. $\mathbf{s}[\neg\phi] = \mathbf{s} \setminus \mathbf{s}[\phi]$
2. $\mathbf{s}[\Diamond\phi] = \{w \in \mathbf{s}\colon \mathbf{s}[\phi] \neq \varnothing\}$
3. $\mathbf{s}[\phi \to \psi] = \{w \in \mathbf{s}\colon \mathbf{s}[\phi][\psi] = \mathbf{s}[\phi]\}$

$\mathbf{s} \models \phi$ iff $\mathbf{s}[\phi] = \mathbf{s}$.

Also: (i) $\mathbf{s}[p] = \{w \in \mathbf{s}\colon w(p) = 1\}$; and (ii) $\mathbf{s}[\phi \wedge \psi] = \mathbf{s}[\phi][\psi]$.

So: $\mathbf{s}[\phi \to \psi] = \mathbf{s}$ iff $\mathbf{s}[\phi] \models \psi$.

Note the difference with $\neg\phi$: here it is complementation but in AS it goes pointwise, testing that $\{w\} \not\Vdash \phi$ for every $w \in \mathbf{s}$. This is what makes it OK to say that $\mathbf{s} \Vdash \Diamond\phi$ iff $\mathbf{s} \not\Vdash \neg\phi$.

This is not unlike a considered and rejected thing in data semantics (DS). See Veltman 1985.

Definition 2 (Entailment choices). $\phi_1, \ldots, \phi_n \models \psi$ iff ...

1. `ttc`: ...for any $\mathbf{s}$: $\mathbf{s} \models \phi_1$ and ...and $\mathbf{s} \models \phi_n$ implies $\mathbf{s} \models \psi$.
2. `utc`: ...for any $\mathbf{s}$: $\mathbf{s}[\phi_1]\ldots[\phi_n] \models \psi$.
3. `mtc`: ...$\mathbf{1}[\phi_1]\ldots[\phi_n] \models \psi$.

$\mathbf{1}$ is the state of ignorance.

$\Vdash_{\text{AS}}$ is preservation of assertability/support/truth at a state. This is the natural and obvious choice and is very close to $\models_{\texttt{ttc}}$.

2 Persistence and (non)monotonicity

$\Vdash_{\text{AS}}$ and $\models_{\texttt{ttc}}$ have structural properties that we might not want. Like: (right) monotonicity.

Definition 3. ϕ is persistent iff $\mathbf{s} \Vdash \phi$ and $\mathbf{t} \subseteq \mathbf{s}$ imply $\mathbf{t} \Vdash \phi$.

Things like $\Diamond p$ aren't persistent: adding information can its destroy the assertability.

Observation 1. $\Vdash$ is monotonic: $\phi \Vdash \psi$ implies $\phi, \chi \Vdash \psi$.

You might expect that entailment involving non-persistent sentences isn't (right) monotonic. This interacts with what are otherwise good-making features of $\Vdash_{\text{AS}}$.

(1) It might be Red or Yellow in the box. $\Diamond(p \vee q)$
So: It might be Red in the box. $\therefore \Diamond p$

(2) It might be Red or Yellow in the box. $\Diamond(p \vee q)$
But it's not Red in the box. $\neg p$
So: It it must be Yellow in the box. $\therefore \Box q$ (and $\not\therefore \Diamond p$)

3 Other structural features

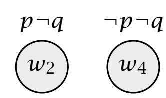
Definition 4. Some entailment properties:

1. `fc`: $\Diamond(\phi \vee \psi) \Vdash \Diamond\phi \wedge \Diamond\psi$
2. `dp`: $\neg\Diamond(\phi \vee \psi) \Vdash \neg\Diamond\phi \wedge \neg\Diamond\psi$
3. `trans`: $\phi \Vdash \psi$ and $\psi \Vdash \chi$ imply $\phi \Vdash \chi$
4. `contr`: $\phi \Vdash \psi$ implies $\neg\psi \Vdash \neg\phi$

Observation 2 (Goldstein)**.** An entailment relation with the above properties also has this one:

5. `exp`: $\Diamond\phi \Vdash \Diamond\psi$

But here's a countermodel in AS: $\mathbf{s} = \{w_2, w_4\}$. Very confusing!



4 Strict-ish conditional

The indicative in AS isn't quite a strict conditional.

Observation 3. Let $\phi \supset \psi = \neg(\phi \wedge \neg\psi)$. Then $\phi \to \psi \ncong \Box(\phi \supset \psi)$.

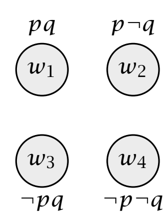
Strict conditionals predict that assserting their negations amounts to asserting that the counterexample to the conditional is possible.

(3) It's not so that if the gardener didn't do it, then it was the butler. It might be the gardener! $\neg\Box(\phi \supset \psi) \cong \Diamond(\phi \wedge \neg\psi)$

Observation 4. $\mathbf{s} \Vdash \neg(\phi \to \psi)$ iff every $w \in \mathbf{s}$ falsifies/denies $\phi \to \psi$. So $\neg(\phi \to \psi) \ncong \Diamond(\phi \wedge \neg\psi)$.

Note that $\{w\} \Vdash \phi \to \psi$ iff $\{w\} \Vdash \phi$ and $\{w\} \Vdash \psi$. So: $\neg(\phi \to \psi) \Vdash \phi$.

Example. Suppose $\mathbf{s} = \{w_1, w_2, w_3, w_4\}$ and note that $\mathbf{s} \nVdash p \to q$ and $\mathbf{s} \nVdash \neg(p \to q)$. But $\mathbf{s} \Vdash \Diamond(p \wedge \neg q)$.

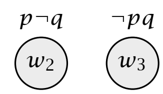


□

(4) It's not so that Yellow must be in the box. It might be Red!
$\neg\Box\phi \cong \Diamond\neg\phi$

Observation 5. $\mathbf{s} \Vdash \neg\Box\phi$ iff every $w \in \mathbf{s}$ falsifies/denies ϕ. So $\Box\phi \cong \phi$ but $\neg\Box\phi \not\cong \Diamond\neg\phi$.

Example. Suppose $\mathbf{s} = \{w_2, w_3\}$ and note that $\mathbf{s} \nVdash \Box p$ and $\mathbf{s} \nVdash \neg\Box p$. But $\mathbf{s} \Vdash \Diamond\neg p$.



$\square$

This comes up in surprising places: US (in)famously doesn't validate modus tollens.

Neither does DS. See (for instance) Veltman 1985; Yalcin 2012; Stojnić 2017; Gillies 2018.

(5) a. If Red is in box #2, then if Blue is in box #2 then Yellow is in box #1. $p \to (q \to r)$

b. It's not so that if Blue is in box #2 then Yellow is in box #1. $\neg(q \to r)$
[Maybe Blue is in #2 and Yellow isn't in #1. $\Diamond(q \wedge \neg r)$]

c. ??So: Red isn't in box #2. $\not\therefore\ \neg p$

d. So: Red might not be in box #2. $\therefore\ \Diamond\neg p$

AS patterns like its relatives if the premise is $\Diamond(q \wedge \neg r)$ but not if it is $\neg(q \to r)$.

(6) a. If the gardener isn't the culprit, then it must be butler. $\neg p \to \Box q$

b. It's not so that it must be the butler. $\neg\Box q$
[The butler might not be the culprit. $\Diamond\neg q$]

c. ??So: the gardener is the culprit. $\not\therefore\ p$

d. So: the gardener might be the culprit. $\therefore\ \Diamond p$

AS patterns like its relatives if the premise is $\Diamond\neg q$ but not if it is $\neg\Box q$.

5 An alternative

Pointwise negation is very strong.

Definition 5. Define both $\Vdash$ and $\dashV$. Possible key clauses:

1. $\mathbf{s} \Vdash p$ as before
   $\mathbf{s} \dashV p$ iff $\mathbf{s} \subseteq [\![p]\!]'$
2. $\mathbf{s} \Vdash \neg\phi$ iff $\mathbf{s} \dashV \phi$
   $\mathbf{s} \dashV \neg\phi$ iff $\mathbf{s} \Vdash \neg\phi$

As is: $\Diamond\phi \nVdash \Box\Diamond\phi$ but $\Diamond\phi \Vdash \Diamond\Box\phi$. This is like DS and not like US. The alternative sketched here would make AS even more like DS.

3. $\mathbf{s} \Vdash \Diamond\phi$ iff $\mathbf{s} \not\dashv\!\!\Vert \neg\phi$
   $\mathbf{s} \dashv\!\!\Vert \Diamond\phi$ iff $\mathbf{s} \Vdash \phi$
4. $\mathbf{s} \Vdash \phi \to \psi$ as before
   $\mathbf{s} \dashv\!\!\Vert \phi \to \psi$ iff $\mathbf{s} \Vdash \Diamond\phi$ and $\mathbf{t}$ blocks ψ

As before $\mathbf{t}$ is the union of worlds in some successor state or other of $\mathbf{s}$ that supports ϕ.
There might be more; I don't know.

Two options for blocking.

Definition 6. $\mathbf{t}$ blocks ψ iff ...

1. ... $\mathbf{t} \dashv\!\!\Vert \psi$.
2. ... $\mathbf{t} \nVdash \psi$.

Observation 6. Suppose $\mathbf{t}$ blocks ψ iff $\mathbf{t} \dashv\!\!\Vert \psi$. Then $\neg(\phi \to \psi) \ncong \Diamond(\phi \wedge \neg\psi)$. Suppose $\mathbf{t}$ blocks ψ iff $\mathbf{t} \nVdash \psi$. Then $\Diamond(\phi \to \psi) \cong \phi \to \psi$.

6 Wrapping up

Three things: (i) interesting paper! (ii) AS is like a gateway drug to dynamic semantics; (iii) try the harder stuff, too.

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