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Conditionals

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1 Introduction: Conditional Information

I want very much that you have the information that the beer is gone. In fact, I want you to take action that requires it. (Let us also stipulate that that action is unavailable (insert your favorite constraint here: policy, prudence, politeness) if there is plenty of beer.) So I say something that gives voice to the state of affairs, beer-wise:

(1) The beer is gone.

I hope that you understand me and take that information on board and then do the right thing. But things won't work out if I'm mistaken about the facts, for then what I am trying to pass off as information isn't that and my hopes will go unfulfilled. Nutshell: successful information exchange depends on the way things are.

That's clear (enough) if the information I aim to get to you is plain (enough) and the linguistic vehicle simple (enough). Conditional information is a useful kind of information and it is no surprise that natural language has canonical ways of expressing it. That is what *if*s — conditionals — are for. So take a case of conditional information exchange where the information is less plain and the linguistic vehicle less simple:

(2) a. If Jimbo is here, then he bought this round.
b. If Jimbo is here, then he might buy this round.
c. If Jimbo were here, then he would buy this round.

The information at stake here is information about what is or might or might not be the case if Jimbo is here, and what would or wouldn't be the case if he had been. Nutshell: successful (conditional) information exchange depends on the way things are but also on the ways things are in various alternative scenarios (the way things are if such-and-such).

Not all conditional information is the same, and this is reflected in differences in conditionals. Take a so-called Adams pair (Adams 1975):

(3) a. If Oswald didn't kill Kennedy, then someone else did.
b. If Oswald hadn't killed Kennedy, someone else would have.

The first conditional is an indicative conditional, saying what is the case if it turns out that Oswald wasn't involved. The second conditional is different. It has distinctive tense/aspect saying what would have been if, contrary to the facts, Oswald hadn't been involved. Such conditionals are counterfactual conditionals, but the name isn't a perfect fit.[1] The types of conditionals are genuinely different since (3a) and (3b) have the same antecedent and the same consequent, but one is true and the other false.

Two not-unrelated bundles of questions will occupy us here, regardless of the type of conditional we are considering. First bundle: What is that conditional information that is conventionally carried by various conditional constructions in natural language? Answering this will involve saying something about a conditional's dependence both on how things are and on how

1 Counterfactuals are sometimes called "subjunctive conditionals", but this is an even worse fit. A natural thought: the "subjunctive" marking goes exactly with counterfactuality. Alas, no, as an example from Anderson (1951) shows:

(i) If Jones had taken arsenic, he would have shown just exactly those symptoms which he does in fact show.

A doctor may well use a conditional like this to argue that what afflicts poor Jones is arsenic poisoning. If counterfactuality means anything like (the speaker is taking it that) the antecedent is false, then the distinctive marking isn't sufficient counterfactuality. Nor is it necessary. In sportscasterese, seemingly run-of-the-mill indicative conditionals are used to convey counterfactual meanings. It's the top of the ninth, the visiting team is down a run but they are down to their last out with a runner — Speedy, as it happens — on second. Slugger hits a double to the gap — surely Speedy should score! — but as Speedy rounds third he trips, falls, and is thrown out. The visitors lose in a shocker. The announcer almost can't believe it. In the aftermath he says:

(ii) If Speedy stays on his feet, they probably win the game.

The announcer isn't confused about how the game ended: he's sure Speedy would have scored and Slugger would likely have been hit in, too. See von Fintel 1998 (and the references therein) for the status of the connection between "subjunctive" marking and counterfactuality.

things might have been. It will also (but not invariably as we will see) involve saying when conditionals are true and when they are false. Second bundle: How do the various conditional constructions in natural language manage to carry that information? Answering this will involve saying something both about how a conditional's meaning arises from the parts of it and something about how a conditional's meaning interacts with and contributes to the meaning of embedded and embedding environments. The bundles are in principle separable, but in practice often enough go hand in hand.[2] The first thing is all about saying what conditionals mean (what semantic values they have) and how things that mean those things are used in well-run conversations (what their pragmatic profiles are). The second thing is all about how *if* interacts with the rest of our language — how what specific conditionals mean is determined by the bits that make them up and about conditional constructions contribute whatever-it-is they mean to embedding environments in which they occur as proper parts.

The aspiration is to at least see where some answers can be found.[3]

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2 There are complications and wrinkles galore. Here are just three, followed by executive decisions about the issues they raise. One: some conditional constructions do not seem to carry conditional information at all.

(i) There are biscuits on the sideboard if you want them. (Austin 1956)

These are so-called biscuit (or relevance) conditionals and are fine things to say but they don't normally (or obviously) express a what conditionals normally (or obviously) do. So let's agree to set them aside. Two: there are ways to conventionally express conditional information in natural language without resorting to *if*. This is especially clear with conditional imperatives:

(ii) a. Be on time or text me!
b. Keep it up and I'll turn this car around!

These do manage to express some conditional kind of meaning (iffiness but *if*lessness!). Still, set them aside. (While we're at it: we will also not have anything to say about conditional imperatives or conditional questions.) And three: Some languages (apparently) lack lexicalized *if*-constructions altogether. We will largely ignore cross-linguistic pressures, too. The hope is that the executive decisions won't distort things (too much).

3 Though this is a survey, it can't (and won't) pretend to be either exhaustive or unopinionated. See Section 9 for references that may provide balance on either horn.

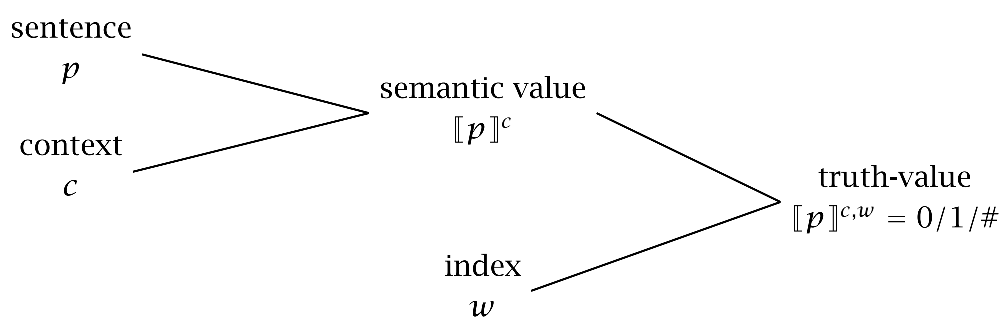

Figure 1 Variable But Simple Semantic Values

2 Preliminaries

Distinguish between (i) conditional sentences (indicative or counterfactual) of natural language and (ii) conditional connectives of some formal language that serves to represent the relevant conditional sentences. The aim is to associate an *if*-in-a-natural-language with an *if*-in-the-formal-language that then gets associated with its semantic value. We will assume that the role of the formal language can be adequately played by a simple propositional language with the usual sentential connectives $(\neg, \wedge, \vee, \supset)$ plus a binary sentential connective $(\textit{if}\ \cdot)(\cdot)$. Context will disambiguate which sort of conditional $(\textit{if}\ \cdot)(\cdot)$ represents.[4] (When the time comes, we will also have use for modal operators of the usual sort like $\Box$ and $\Diamond$ and perhaps a few other things.) This two-step route to assigning meanings to conditionals isn't obligatory and is in-principle dispensable (assuming that the mapping from natural language to the formal language (we won't fuss with this one) and then the mapping from the formal language to the universe of meanings (we will fuss with this one) are well-behaved). But it does make for a clearer view of the landscape.

A (formal) language is only useful if it is interpreted. Whether or not truth-in-English has been achieved by an utterance of

(4) Jimbo has to be washing the dishes.

4 Whether conditionals in natural language can be represented by a binary conditional connective in a regimented intermediate language is also, as we'll see, up for grabs.

depends on the facts — how things actually are at the world where it is uttered. It also depends on the state of the conversation when it is uttered: for instance, whether what is being claimed is a claim about Jimbo's obligations or about what we know about his current activities. And it also depends on how things are in various other situations: for instance, whether at those relevant situations the prejacent *Jimbo is washing the dishes* is true. So truth-in-English is sensitive to both a context of utterance and an index of evaluation. One of the things we're after is a way of systematically saying when a sentence p is true in a context at a world.[5] So let's suppose that semantic values (whatever they are) determine truth values at points of evaluation with respect to contexts. We will not need to say just what contexts are. It will be enough to carve out what role they play in the set-up. Similarly, our indices will be worlds, but the most we need to know about them is that they are the kinds of things at which sentences are true or false.[6] The set-up is summarized in Figure 1: a sentence p in a context c gets associated with a semantic value $[\![p]\!]^c$, which combines with an index w to deliver a truth-value (if the sentence has one) of the sentence at that world in that context, $[\![p]\!]^{c,w} = 1$ or $[\![p]\!]^{c,w} = 0$ or $[\![p]\!]^{c,w} = \#$ (as the case might be). These aren't entirely innocent assumptions, but we can start here and suspend them when it suits us.

Theories of conditionals are constrained by the patterns of intuitive entailment they participate in. So what-intuitively-entails-what is important data and we want to explain its patterns as best we can. For conditionals this might tie what $(\mathit{if}\ \cdot)(\cdot)$ might mean very tightly to what we can and can't say about entailment. For instance, take the basic deduction theorem.

Deduction Theorem $X, p \models q$ iff $X \models (\mathit{if}\ p)(q)$

Taking this as a constraint, what we say about $(\mathit{if}\ \cdot)(\cdot)$ impacts what we can say about $\models$ and vice versa. (This holds for various weakenings of the deduction theorem too, wherein the weakness lies in how the set of premises A and the premise p are combined.) The point is just that "entailment" is as much part of the theoretical machinery as is anything. So it's up for grabs whether what we say about *if*s bends to the will of entailment or whether

5 Lewis (1980) argued that contexts and indices are both needed and that neither can do the work of the other.

6 For now. Later, when we flirt with various dynamic theories, all we will need to know about them is that they are the kind of thing that atomic sentences (of our intermediate language) are true or false at. If an atomic a is true (false) at w, we'll say that $w(a) = 1$ ($w(a) = 0$).

what we say about entailment bends to the will of the *if*s. There is very little on stone tablets. That said, let's try to skirt issues about entailment when we can.

The simplest (and in a precise sense the weakest) conditional is the material conditional: $p \supset q$ is true iff either p is false or q is true. Accordingly, the simplest (and in a precise sense the weakest) theory of *if* takes it to simply be the material conditional.[7]

Definition 1 (Horseshoe Theory)**.** Conditionals are material conditionals:

$$[\![(\textit{if}\ \ p)(q)]\!]^{c,w} = 1 \text{ iff either } [\![p]\!]^{c,w} = 0 \text{ or } [\![q]\!]^{c,w} = 1.$$

This is, so far, neutral about whether the target conditionals are indicative or counterfactual. That matters and we will return to it.

In any case, this treats conditionals as truth-functional (and, by the way, context-invariant): the truth-value of a conditional at a world is entirely determined by the truth-value of its antecedent and consequent at that world. In fact, this is the only truth-functional option available for the conditional.

Fact 1. If $(\textit{if}\ \ \cdot)(\cdot)$ is truth-functional then $(\textit{if}\ \ \cdot)(\cdot) = \supset$.

Here's why. Suppose p = *The die came up six* and q = *the die came up even*. Our theory has to then render $(\textit{if}\ \ p)(q)$ true no matter how the roll came out. So, in particular it's true if it came up six (antecedent true, consequent true), if it came up four (antecedent false, consequent true), and if it came up three (antecedent false, consequent false). The rub is that since by hypothesis $(\textit{if}\ \ p)(q)$ only depends on the (actual) truth values of p and q, any p and q with the same truth-values can be substituted in for them and the resulting conditional has to still be true. So: if a truth-functional conditional has a false antecedent or true consequent it is true. Now assume some conditional is false. It can only be because it has a true antecedent and false consequent. Given truth-functionality, this then holds for all conditionals. So: if a truth-functional conditional is false it has a true antecedent and false consequent.

As a theory of counterfactuals, this is an obvious non-starter. Conceptually, counterfactuals — the real deal ones with false antecedents — ask us to consider other, non-actual ways things might have been. But a truth-functional theory says that such considerings can't be relevant. The empirical

7 So-called because the material conditional is sometimes symbolized by the horseshoe $\supset$.

coverage is also terrible. At least some counterfactuals with false antecedents are (contingently) true and thus worth arguing about. The material conditional rules that out.

(5) a. If Alex had come to the party, she would have arrived before 8.
b. If Alex had come to the party, she wouldn't have arrived before 8.

These conditionals cannot both be true, and speakers using them seemingly disagree. That is not what you'd expect if counterfactuals were horseshoes.

When it comes to indicative conditionals things are different: the horseshoe isn't widely adopted, but it is not without defenders. The main difficulty is that the material conditional is, from a logical point of view, weak. Taking indicatives to be horseshoes thus predicts that there are more entailments to conditionals than there seems to be. Among them: the paradoxes of material implication.

(6) a. Carl came alone.
??So: if Carl came with Lenny, neither came.
b. Billy got here first.
??So: if Alex got here before Billy, Billy got here first.

These don't strike as entailments even though the truth of either $\neg p$ or q at w secures the truth of $p \supset q$ at w.

The thing that has to be said is that while these are genuine entailments, there are pragmatic reasons — derived from how conditionals and surrounding sentences are reasonably and appropriately deployed in conversation — why they strike us as weird. For instance: the conditional conclusions in (6) are weird because in each case a speaker in the position to assert the unconditional premise has no use for (and hence would mislead by using) the logically weaker conditional conclusion. So there is a clash between an implicature of the conclusion and the initial premise.[8]

Explaining away unwanted entailments by appeal to implicatures is tricky since this strategy has nothing to say about conditionals that occur unasserted in embedded environments. But conditionals occur in such

8 This is Grice's (1975) strategy and is taken up and extended by Lewis (1976). Another pragmatic defense, developed by Jackson (1991), says that when you use an indicative $(\textit{if } p)(q)$, it conventionally implicates that your credence in its truth (that is, the truth of $p \supset q$) conditional on p is high. (Lewis, in the "Postscript" to his 1976, drops the conversational story and instead goes for a slight variant of Jackson's.) There is little independent evidence in favor of this stipulated conventional implicature.

environments and when they do the horseshoe theory makes some unhappy predictions. Negated indicatives are a case in point. The issue is that since material conditionals are so weak, their negations are correspondingly strong.

(7) It's not so that if the gardener didn't do it then the butler did.

I can be signed-up for (7) without being signed-up for the truth of *the gardener didn't do it*. After all, it might have been the driver. Maybe we need a pragmatic defense of our pragmatic defense. Perhaps what we have in (7) is a denial of a conditional rather than an assertion of a negated conditional, and so the negation is not a negation. I doubt it since the issue can be pushed where the negated conditional itself occurs embedded and thus unasserted and thus not open to this defense. For instance: the argument in (8) is a disaster but predicted to be an entailment by the horseshoe.

(8) If there is no god, then it is not the case that if I pray, my prayers will be answered.
I don't pray.
??So: there is a god.

Is there another pragmatic defense to save these pragmatic defenses of the earlier pragmatic defense? It's possible, but this rescue is quickly becoming a wheels-within-wheels situation.

3 Strict Conditionals

The material conditional is extreme in its myopia: only how things are at w matter to the truth of a material conditional at w. At the other end of the spectrum lie strict conditionals: these survey all possibilities. That is a quantificational claim and so, as with a lot of quantificational claims, this one may be restricted.[9] We will care about the *if*-relevant worlds in a context c. A strict conditional is a (restricted) universal quantifier, saying that all the *if*-relevant possibilities at which the antecedent is true are possibilities at which the consequent true. When the only job of contexts is to provide such worlds — as it is now — let's simply identify a context c with the selection function that delivers the *if*-relevant for each world w.

9 *All the beer is gone!* often does not mean that the universe is out of beer but something more modest like the relevant beer supply is out.

Definition 2 (Strict Conditionals)**.** Let $c(w)$ be the set of *if*-relevant worlds at w (in c).

$$[\![(if\ p)(q)]\!]^{c,w} = 1 \text{ iff if } v \in c(w) \text{ and } [\![p]\!]^{c,v} = 1 \text{ then } [\![q]\!]^{c,v} = 1.$$

This is equivalent to saying that $(if\ p)(q)$ is true at w with respect to c iff the material conditional $p \supset q$ is true at every world in $c(w)$.

What does it take to be an *if*-relevant world? We haven't said (that's by design). Thus depending on what we say about what it takes for a world to be an *if*-relevant world at w (that is, depending on what we say about the function c given an argument w), we get a different strict conditional.

Here are some possibilities (not exhaustive):

- For any w, only w ever matters (for every w: $c(w) = \{w\}$). The resulting strict conditional is the material conditional.
- For any w, all worlds (unrestricted!) always matter (for every w: $c(w) = W$). The resulting strict conditional is strict implication (true iff the antecedent entails the consequent).
- For any w, all worlds compatible with what is known in w matter (for every w: $c(w) = \{v\colon$ if X is known at w then $v \in X\}$). The resulting strict conditional is a sort of epistemic strict conditional.
- For any w, all worlds similar to w to fixed degree d matter (for every w: $c(w) = \{v\colon$ v is similar to w to at least degree $d\ \}$). The resulting strict conditional is a sort of similarity-based strict conditional.

Notice that for each such $c(w)$ there is a corresponding (restricted) necessity operator $\Box_{c(w)}$ and a strict conditional $(if\ p)(q)$ with respect to $c(w)$ amounts to claiming that the corresponding material conditional $p \supset q$ is $\Box_{c(w)}$-necessary. The strictness of two strict conditionals can be compared: if the set of *if*-relevant worlds (at a world) for one strict conditional is included in the set of *if*-relevant worlds for a second strict conditional, then since the second quantifies over more possibilities it is a stricter strict conditional.

Fact 2. Suppose $(if_1\, p)(q)$ is a strict conditional w.r.t. $c_1(w)$ and $(if_2\, p)(q)$ is a strict conditional w.r.t. $c_2(w)$. If $c_1(w) \subseteq c_2(w)$ then $(if_2\, p)(q)$ implies $(if_1\, p)(q)$.

This way of thinking about conditionals treats them as restricted necessity operators, saying that their consequents are true throughout a given set of antecedent worlds. Since we can probe necessity operators by examining the properties of the functions which determine the sets of worlds they quantify over, the same is true of restricted necessity and so of strict conditionals. For instance: suppose that every world is relevant to an *if* at that world. That means that c is reflexive: for every w it is the case that $w \in c(w)$.

Fact 3. c is reflexive iff $(\textit{if}\ p)(q)$ implies $p \supset q$.

Two comments. First: this amounts to saying that *if* is on the spectrum of logical strength, occupying a place no weaker than the material conditional. (Since $c(w)$ is always a set of worlds we have built-in that strict implication is at least as strong as *if*.) Since the material conditional together with its antecedent entails its consequent (on any sensible notion of "entails") it follows that being stricter than the material conditional means going in for some version of modus ponens. We will come back to this. Second: this follows straightaway from the fact that $(\textit{if}\ p)(q)$ with respect to $c(w)$ is equivalent to $\Box_{c(w)}(p \supset q)$ and the fact that in modal logic reflexive accessibility relations (or functions or spheres or whatever) correspond to the validity of instances of the T axiom $\Box p \supset p$.

Another sort of property is the package of import/export equivalences.

Import/Export $(\textit{if}\ p \wedge q)(r) \dashv\vDash (\textit{if}\ p)((\textit{if}\ q)(r))$

Some potential import/export pairs for indicatives:

(9) a. If Carl is away, then if Lenny is away, then Sector 7G is empty.
b. If Carl is away and Lenny is away, then Sector 7G is empty.

And for counterfactuals:

(10) a. If Alex had come to the party, then if Billy had come to the party it would have been great fun.
b. If Alex had come to the party and Billy had come to the party, it would have been great fun.

Once we are thinking of conditionals as restricted necessity operators, import/export reads a lot like some sort of constraint on introspection. That's close to right.[10]

10 This discussion follows the discussion in Kaufmann & Kaufmann 2015 and to some extent some of the discussion in van Benthem 2001: §3.2. As we'll see this provides a modal

Fact 4. $(if\ \cdot)(\cdot)$ supports the right-to-left direction of Import/Export iff c is "shift reflexive". That is: iff $v \in c(w)$ implies $v \in c(v)$. It supports the left-to-right direction iff c is "shift coreflexive". That is: iff $v \in c(w)$ and $v \in c(u)$ implies $u = v$.[11]

Together (shift) reflexivity/coreflexivity — that is, both halves of import/export — impose a very strong requirement: that whenever a world is *if*-relevant at w that world is related to itself and to no other. (This property is what Kaufmann & Kaufmann call "shift identity".) That seems to be a weird requirement.

Matters get immediately worse. Combining reflexivity of the assignment of *if*-relevant possibilities with shift reflexivity/coreflexivity means that that assignment is an island function: every world is an *if*-relevant island to a conditional at that world (for every w: $c(w) = \{w\}$).

Fact 5. $(if\ \cdot)(\cdot)$ implies the corresponding material conditional and supports Import/Export iff for every w: $c(w) = \{w\}$.

As we saw: a strict conditional over such island sets of *if*-relevant possibilities is just the material conditional. So any conditional comparable to and no weaker than material implication that supports Import/Export must be the material conditional. Such "collapse" arguments serve as clear maps to the landscape of conditionals (especially indicatives). We will return to this theme.

There are other properties that strict conditionals (of whatever strictness) have, irrespective of the properties of the sets of *if*-relevant possibilities. I will mention three.

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perspective to Gibbard's (1981) argument that reduces any "propositional" operator meeting a few minimal assumptions to the material conditional.

11 To see that if $v \in c(w)$ then $v \in c(v)$ implies the right-to-left direction of import/export: assume that $(if\ p)((if\ q)(r))$ is true at w w.r.t. c and consider any $v \in c(w)$ such that $[\![p \wedge q]\!]^{c,v} = 1$. (We want to show that $[\![r]\!]^{c,v} = 1$ and hence that $(if\ p \wedge q)(r)$ must be true at w w.r.t. c.) Since $[\![(if\ p)((if\ q)(r))]\!]^{c,w} = 1$ it follows that all p-worlds in $c(w)$ are worlds where $(if\ q)(r)$ is true. Hence since $[\![p]\!]^{c,v} = 1$, it follows that $[\![(if\ q)(r)]\!]^{c,v} = 1$. So all q-worlds in $c(v)$ must be worlds where r is true. Since if $v \in c(w)$ then $v \in c(v)$ and $[\![q]\!]^{c,v} = 1$ it thus follows that $[\![r]\!]^{c,v} = 1$, as desired. (For the other direction(s): it's generally easier to go by showing how if c isn't constrained (e.g., if it's not in general true that $v \in c(w)$ implies $v \in c(v)$) then you can use the witnessing worlds to counterexample the relevant logical principle. Providing full proofs of these correspondences is left as a rainy day exercise for the reader.)

Antecedent Strengthening $(if\ p)(q) \models (if\ p \wedge r)(q)$ for any r

Transitivity $(if\ p)(q), (if\ q)(r) \models (if\ p)(r)$

Contraposition $(if\ p)(q) \models (if\ \neg q)(\neg p)$

Fact 6. Any strict conditional supports antecedent strengthening, transitivity, and contraposition.

The reason for all three properties is simple: strict conditionals are universal quantifiers, saying all the *if*-relevant possibilities are of a certain sort. And the thing about *all*: all-claims are downward monotone, support transitive inferences, and contrapose.[12] Some quick Venn diagrams will convince the unmoved.

These properties push away from strict conditional analyses of (especially) counterfactuals, and (as we'll see) predicting that these patterns are patterns of non-entailment is a highlight of the classic variably strict theory developed initially by Stalnaker (1968, 1984) and Lewis (1973). Some concrete counterexamples:

(11) a. If Sophie had gone to the parade, she would have seen Pedro dance.
??So: If Sophie had gone to the parade and been stuck behind someone tall, she would have seen Pedro dance.

b. If Hoover had been a communist, he would have been a traitor.
If Hoover had been born a Russian, he would have been a communist.
??So: If Hoover had been born a Russian, he would have been a traitor.

c. If it had rained, it wouldn't have poured.
??So: If it had poured, it wouldn't have rained.

In fact the indicative counterparts here also make trouble for strict conditional accounts of indicatives. Set this aside for now.

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12 That is:

i. *All As are Cs* $\models$ *All Bs are Cs* whenever $B \subseteq A$;

ii. *All As are Bs*, *All Bs are Cs* $\models$ *All As are Cs*;

iii. *All As are Bs* $\models$ *All non-Bs are non-As*.

4 Variably Strict Conditionals

We have come this far without saying whether the *if*s at stake are indicatives or counterfactuals. Let's change that by focusing on counterfactuals (though, not quite exclusively). The classic account of counterfactuals, and by far the account that remains dominant, treats them as variably strict conditionals (Stalnaker 1968, Lewis 1973).

Look again at (11a). This is problematic for strict conditionals since it seems eminently possible that (i) had Sophie gone she would have seen Pedro dance, and (translating to our formal language) so (ii) the conditional $(if\ p)(q)$ is true. Though, admittedly, (iii) not if she had been stuck behind someone tall. So, (iv) the counterfactual $(if\ p \wedge r)(q)$ is false and, it seems, (v) the contrary conditional $(if\ p \wedge r)(\neg q)$ is true. Strict conditionals don't seem to have the needed flexibility to allow both (ii) and (v). Put another way: sequences of counterfactuals like $(if\ p)(q)\,;(if\ p \wedge r)(\neg q)$ seem like they can be consistent — both conditionals can be non-vacuously true at the same time. Since that's not true of strict conditionals, Lewis (1973) argued on this basis that counterfactuals can't be any strict conditional no matter the level of strictness.

Some examples:

(12) a. If the USA threw its weapons into the sea tomorrow, there would be war;
b. But if the USA and the other nuclear powers threw all their weapons into the sea tomorrow, there would be peace.

(13) a. If you had been standing a foot to the left, you would have been killed;
b. But if you had (also) been wearing your hard hat, you would have been alright.

(14) a. If Sophie had gone to the parade, she would have seen Pedro dance;
b. But if Sophie had gone to the parade and been stuck behind someone tall, she would not have seen Pedro dance.

With a little ingenuity, it seems each sequence can be extended for as long as you like, with each successive conditional having an ever-stronger antecedent and the consequent flipping from negated to not. Such sequences are called

Sobel sequences.[13] Analyzing any Sobel sequence as a sequence of strict conditionals, no matter the strictness, won't work and the reason is just that strict conditionals support antecedent strengthening.

Lewis's conclusion is that there is no level of strictness such that in a given situation counterfactuals are strict conditionals of that strictness. Instead, they are variably strict conditionals: they are more or less strict depending on the strengths of their antecedents. Intuitively (with a not insubstantial additional assumption that we'll return to) a counterfactual like (14a) says that all of the possibilities most similar to ours but in which Sophie went to the parade are possibilities wherein she witnessed Pedro's dancing. The "thinned" (14b) on the other hand says something about all of the possibilities most similar to ours but in which Sophie went to the parade and was stuck behind someone tall (namely that in those worlds she missed out on the dancing). These are compatible so long as the (modally) nearest go-to-the-parade possibilities do not include the nearest go-to-the-parade-and-get-stuck-behind-someone-tall possibilities.

To make this less impressionistic: rather than simply associating with each world w a set $c(w)$ of *if*-relevant possibilities — possibilities relevant regardless of the *if* — we want to look at the nearest or most similar possibilities to w that make a particular antecedent true. So we need some (context dependent) measure of relative similarity or nearness or closeness of worlds to w. We'll write it this way: $\leq_w$, with the idea that if $v \leq_w u$ then v is at least as close to w as u is.[14] Officially we are thinking that the context c determines such an ordering for each world, but we won't say how that happens and will generally suppress mention of c. Even before saying anything about this relation we know how the story will appeal to it.

Definition 3 (Variably Strict Conditionals)**.** Conditionals are variably strict conditionals:

$$[\![(\textit{if } p)(q)]\!]^{c,w} = 1 \text{ iff if } v \text{ is a } \leq_w\text{-minimal } p\text{-world then } [\![q]\!]^{c,v} = 1$$

where a world v is a $\leq_w$-minimal p-world iff there is no p-world u such that $u <_w v$.

13 Lewis credits Sobel for pointing them out.

14 To be sure: the spatial metaphor here is, in fact, a metaphor. We'll return to this at the end of this section.

Given this set-up, there are some straightforward things to be said about the nearness ordering. For one thing, it has to actually be an ordering. For another thing, it should respect the fact that each world is an awful lot like itself and so pretty close to itself.

Definition 4 (Relative Similarity)**.** For any worlds w, v let $\leq_w$ be such that:

i. (Ordering Assumption) $\leq_w$ is a connected and transitive relation over W.

ii. (Centering Assumption) w is minimal in $\leq_w$: if $v \leq_w w$ then $w = v$.

Together Definition 3 and Definition 4 form the bare-bones of the story.[15]

There are other notable assumptions we might add to it by imposing further constraints on the ordering (for any proposition X and world w):

iii. (Limit Assumption) X has at least one $\leq_w$-minimal world.

iv. (Uniqueness Assumption) X has no more than one $\leq_w$-minimal world.

Stalnaker's (1968) set-up makes both of these additional assumptions, while Lewis's (1973) makes neither. Some of the properties are directly reflected in logical principles. For instance:

Conditional Excluded Middle $\models (\textit{if } p)(q) \vee (\textit{if } p)(\neg q)$

Fact 7. If $\leq_w$ satisfies both the Limit Assumption and the Uniqueness Assumption then $(\textit{if } \cdot)(\cdot)$ supports conditional excluded middle.

The additional assumptions on the orderings are separable. I won't be assuming uniqueness (for the most part) but will make the limit assumption — indeed, above in Definition 3 I already made the limit assumption.[16]

15 Sort of. We might go instead with weak centering instead of centering: $w \leq_w v$ for every v. This requires that w is always among the closest worlds to w, but allows that other worlds might be equally close. There aren't decisive reasons for favoring weak centering over centering since both deliver modus ponens for the counterfactual. Or we might (as Pollock (1976) does) weaken the ordering assumption by permitting incomparabilities as well as ties in the ordering.

16 Lewis offered this reason against the limit assumption. Take a counterfactual like this:

(i) If Alex had been a little taller, she would have played basketball.

It is easy to see why this approach is attractive. Take our Sobel sequences (12)–(14). If the conditionals are variably strict then no wonder they are consistent: in each case the first conditional takes us to the nearest possibilities in which a simple antecedent is true, but the second one takes us to a set of possibilities in which in addition some other thing is true. Since these need not be the same (indeed, aren't) the two conditionals end up talking about different possibilities.

The same goes for the problematic entailments that strict conditionals support.

Fact 8. Variably strict conditionals do not support antecedent strengthening, transitivity, or contraposition.

Variably strict conditionals don't validate any of these patterns and that is in their favor as a theory of counterfactuals in natural language.

Given our assumptions we could easily re-tell the story in terms of sets of (antecedent specific) *if*-relevant worlds rather than the (family of) nearness or similarity orderings by, for a given world w and antecedent p, collecting up the set of worlds no further from w than the nearest p-world.

Definition 5. $c(w) = \lambda p.\{v : \text{ if } u \text{ is a minimal } p\text{-world in } \leq_w \text{ then } u \not<_w v\}$

One thing to notice is that centering (actually, weak centering) translates to the constraint that for any w and (consistent) p it is the case that $w \in c(w)(p)$. That means that counterfactuals would support modus ponens, and that seems strange since counterfactuals carry some sort of signal that their antecedents are false and those aren't the kinds of things that modus ponens comes up for.[17] Still, I think this is as is it should be.

(15) a. Alex: If my keys had been in the drawer, I would have seen them.
b. Billy: No, look: they were in the drawer under the mail and you missed them!

No matter Alex's height, there is no world closest to ours in which Alex is just a little taller. Take any world v where Alex's height is h_v where that is a bit taller than her height $h_@$ here. There is another world u such that her height h_u is intermediate. That's just how height works. The argument is not quite decisive, though, since giving up the limit assumption means giving up some pretty desirable features, too. (See, especially, Pollock 1976, Herzberger 1979.) And see Swanson 2012 for a discussion of preserving conditional excluded middle without the Limit Assumption.

17 Just what kind of signal? There is a debate. It won't matter too much for us, so set it aside. But see Stalnaker 1975, Veltman 1976, and von Fintel 1998.

Billy's denial here makes sense because the truth of the counterexample $(p \wedge \neg q)$ at a world is always sufficient for the falsity of the counterfactual $(\mathit{if}\ p)(q)$ at that world. And that is what (weak) centering imposes.

So we have reflexivity. Since variably strict conditionals are stronger than material conditionals (i.e., the sets of relevant *if*-worlds aren't always trivial islands) we know that variably strict conditionals can't validate import/export.

Fact 9. Variably strict conditionals do not support import/export.

Import/export would require that the closest q-worlds to the closest p-worlds are the same as the closest $(p \wedge q)$-worlds. Counterexamples (of the formal stripe) are easy to construct. For instance: suppose we have just the worlds w, x, y, z where p is true at x and y while q is true at y and z. Now, let $\leq_w$ be the ordering $w \leq_w x \leq_w y \leq_w z$. Thus the ($\leq_w$-)nearest $(p \wedge q)$-world to w is y. Finally, let $\leq_x$ be such that $z <_x y$. Thus the ($\leq_x$-)nearest q-world to the ($\leq_w$-)nearest p-world to w isn't y.

As we saw with conditional excluded middle (and the limit assumption) properties of the nearness relations (metaphor!) make for differences in the properties that an analysis of conditionals based on them has. But disagreements about the properties of the nearness relation can seem to be bigger than they in fact are. Assume for simplicity that the space of worlds is finite. Then there are two natural decision points. First decision point: does the ordering permit ties? That is, is the case that two distinct worlds u and v can be tied in closeness or similarity to a world w? Adopting the uniqueness assumption prohibits ties. Second decision point: must all worlds always be comparable? That is, (for any w, u, v) is it the case that either $u \leq_w v$ or $v \leq_w u$? In principle these are separate questions, but in practice the dividing lines are whether ties and incomparabilities are both prohibited (Stalnaker), whether ties are permitted but incomparabiltities are not (Lewis), or whether both ties and incompatibilities are permitted (Pollock).

But, as Lewis (1981) showed, there is less disagreement here than appears. Suppose our ordering permits neither ties nor incomparabilities. That's true for any <u>fixed</u> and <u>determinate</u> ordering and, for all that we've said so far, it's possible that ordinary contexts don't always determine unique orderings. In such contexts there are, let's say, multiple admissible orderings. Then supervaluating over the admissible orderings gives determinate truth-conditions for simple conditionals just like we would get from a theory that permits ties but not incomparabilities.[18] And similarly for the second choice point. A theory that permits ties but not incomparabilities but allows that there may be a range of admissible orderings in any given context is, in this same sense, equivalent to a theory like Pollock's which permits both ties and incomparabilities.

There is one more set of equivalences between theories to round out this part of the discussion. One way to make sense out of the idea of conditional information is embodied by (some version of) the Ramsey Test.

Ramsey Test $(\textit{if } p)(q)$ is true/accurate/acceptable in a situation iff q is true or accurate or acceptable in that situation-plus-the-information-that-p.

Depending on what we say about "true/accurate/acceptable", "situations", and "situation-plus-the-information-that-p" we get different theories.[19]

There are ways of filling in those details for counterfactuals. Most prominently among them are versions of premise semantics.[20] The idea is that (relative to a context, just as with the orderings) we associate with each world a set of propositions true at that world, a premise set P_w at w, and use that as the factual background to assessing a counterfactual's truth at w. Just like with similarity orderings, this assignment depends on context though we won't say just how. A conditional $(\textit{if } p)(q)$ is true at a world iff every way of consistently adding the information that p to those premises gives us a derived or subordinate set of premises that entails q.

18 That is: rendering a conditional determinately true iff if it's true according to each admissible order. Stalnaker (1984) develops this approach in his defense of conditional excluded middle.

19 The famous footnote: "If two people are arguing 'If p, will q?' and are both in doubt as to p, they are adding p hypothetically to their stock of knowledge and arguing on that basis about q ..." (Ramsey 1929/1990: 155). The adding this involves can't be feigned or hypothetical belief.

(i) If my students are cheating in class, then I will not discover it (because they're so clever).

The conditional is (let's say) true even though the state I get into by feigning belief in the antecedent won't be one in which I believe the consequent. (Thomason is credited with this observation in van Fraassen 1980.) The right way to understand augmenting for the Ramsey test is restricting a body of information (my belief state or the contextually relevant information or whatever) by the content of the antecedent. That's different.

20 The (independently developed) classics: Kratzer 1981b and Veltman 1976, developed and extended in (respectively) Kratzer 1989 and Veltman 2005.

Let's say things a little more precisely (still sticking to the assumption that W is finite) so we can say how this set-up relates to the variably strict picture.

Definition 6 (Premise Sets)**.** For every world w there is a set P_w of propositions such that:

i. (Exhaustive Assumption) Every world belongs to some proposition in P_w ($\bigcup P_w = W$).

ii. (Centering Assumption) Every member of P_w is true at w and w only ($\bigcap P_w = \{w\}$).

For any proposition X let $\mathbf{P}_w^X$ be the set of maximal X-consistent subsets of P_w.[21]

The first assumption plays the role of the ordering assumption for variably strict conditionals (our orderings left out no worlds) and the second assumption plays the role of centering (naturally enough).

Counterfactuals ask us what follows from the minimal ways of making room for the antecedent. So we look to the maximal p-consistent subsets of our premise set and see what follows from them.

Definition 7 (Conditionals and Premise Sets)**.** Conditionals are minimally revised premise set entailments:

$$[\![(\mathit{if}\ p)(q)]\!]^{c,w} = 1 \text{ iff if } P_w^* \in \mathbf{P}_w^{[\![p]\!]} \text{ then } P_w^* \text{ entails } [\![q]\!].$$

Lewis (1981) showed that such a theory is equivalent to a theory based on orderings that permits ties and incomparabilities between worlds. There are ways of turning premise sets into an ordering and the result of doing that gives us an ordering that says a counterfactual is true iff our original premise set did. (Going the other way, every ordering can be derived in this way from some premise set, with the same result.) It is the deriving of orderings from premise sets that is key:

Fact 10. A premise set P_w induces an ordering $\leq_w^P$ where $u \leq_w^P v$ iff every proposition in P_w true at v is true at u (that is: iff $\{X \in P_w\colon u \in X\} \subseteq$

21 A subset P_w^* (also a premise set at w!) of P_w is X-consistent iff $\bigcap P_w^* \cap X \neq \emptyset$. Such a subset is maximal iff it is contained in no other such subset of P_w.

$\{X \in P_w : v \in X\})$. A conditional $(if\ p)(q)$ is true with resect to P_w iff it is true with respect to $\leq_w^P$.

So premise semantics and ordering semantics can both be sensibly seen as different but broadly equivalent implementations of the variably strict theory.[22]

5 Counterfactual Dynamics

Taking counterfactuals to be variably strict conditionals is the industry standard. But it is open to challenge.[23] A lot of weight has been put on the basic logic that variably strict conditionals deliver. But things are more subtle, and way more interesting, than they at first appear.

Look again at our examples of antecedent strengthening gone wrong (12)–(14). These bear important weight in the argument against taking counterfactuals to be any strict conditional. If they were any strict conditional, each pair would be incompatible but they are as normal as counterfactual discourse ever gets. But as normal as they are, they are dramatically not that — indeed they seem contradictory — when issued in reverse order as reverse Sobel sequences.[24] Look at the difference with Lewis's example. Here is the (a)→(b) direction we saw before:

(12) a. If the USA threw its weapons into the sea tomorrow, there would be war;
b. But if the USA and the other nuclear powers threw all their weapons into the sea tomorrow, there would be peace.

22 There is nothing in the premise semantics set-up that mentions nearness or similarity or anything like that. So to argue that the nearness metaphor in the Lewis-Stalnaker set-up has to be more than a metaphor we need an argument that the corresponding premise sets somehow encode (by what is lumped together and what isn't) nearness or similarity information. I don't know what that would look like.

23 One sort of challenge that we won't discuss is whether there is any substantive notion of similarity that plays the role that some versions of the variably strict theory carve out for it. There are different threads one can tug here. One begins with Fine 1975 and (what can be read as) Lewis's (1979a) reply. This thread is neatly discussed in Bennett 2003. Another thread: similarity is the wrong sort of thing. This argument comes from the premise semantics corner (Tichy 1976, Kratzer 1989, Veltman 2005). And a third thread challenges that there can be no such thing as all-in similarity because there is no sensible way of aggregating the comparisons that the relevant theories say must be aggregated (Morreau 2010).

24 von Fintel (2001) credits Irene Heim with the observation.

And the reversed (b)→(a) direction:

(16) ??If the USA and the other nuclear powers threw all their weapons into the sea tomorrow, there would be peace; but if the USA threw its weapons into the sea tomorrow, there would be war.

Another of our (a)→(b) examples and its reversed (b)→(a) counterpart:

(14) a. If Sophie had gone to the parade, she would have seen Pedro dance;
b. But if Sophie had gone to the parade and been stuck behind someone tall, she would not have seen Pedro dance.

(17) ??If Sophie had gone to the parade and been stuck behind someone tall, she would not have seen Pedro dance; but if Sophie had gone to the parade, she would have seen Pedro dance.

There's no doubting the asymmetry here: unfolding a Sobel sequence (and so a counterexample to antecedent strengthening) from (a)→(b) is one thing, reversing the order from (b)→(a) is another thing entirely. But, from the variably strict point of view, once a counterexample to antecedent strengthening always a counterexample. Fiddling with the order in which we find the nearest worlds in which the USA throws its weapons into the sea tomorrow (maybe not so close) versus finding the nearest worlds in which the USA and all the other nuclear powers throw their weapons into the sea tomorrow (even further away) shouldn't change the fact that the sets we get are different in those different cases. This is puzzling.

The argument against treating counterfactuals as strict conditionals that is based on Sobel sequences isn't airtight. Sobel sequences show that counterfactuals aren't strict conditionals so long as the level of strictness never changes as counterfactual discourse unfolds. So there's an unargued for assumption. Reverse Sobel sequences give us a reason to explore life without it.[25]

I will sketch a version of the strict conditional theory initially developed by von Fintel (2001) and then (not quite conservatively) extended in Gillies 2007. There are three ingredients to the story.[26]

Context keeps track of what worlds are counterfactually relevant.[27] Begin with a system of spheres around a world w: a nested set of sets of worlds $c(w)$ such that $\{w\} \in c(w)$ and $\bigcup c(w) = W$. Assume that context keeps track of such a set and that it determines the counterfactually relevant worlds.

Definition 8. The default hyperdomain at w in c is $\pi_c^0 = c(w)$. Given a hyperdomain π_c the domain of counterfactually relevant worlds (a.k.a. the modal horizon) s_{π_c} is the $\subseteq$-minimal domain in π_c.

So the default modal horizon at w is $s_{\pi_c^0} = \{w\}$. (When the context is clear (or not relevant since the counterfactual domains are the only things we are keeping track of here) we will omit reference to c.)

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25 Lewis saw the loophole in his argument but didn't take it seriously:

> It is still open to say that counterfactuals are vague strict conditionals ... and that the vagueness is resolved — the strictness is fixed — by very local context: the antecedent itself. That is not altogether wrong, but it is defeatist. It consigns to the wastebasket of contextually resolved vagueness something much more amenable to systematic analysis than most of the rest of the mess in that wastebasket. (Lewis 1973: 13)

26 The version we will look at says that the meaning of counterfactuals can be factored inot the context-changing part of meaning and the content-assigning part of meaning. The version in Gillies 2007 is dynamic all the way, based on an argument that (i) *might*-counterfactuals exhibit the same sort of context-affecting as *would*-counterfactuals with strengthened antecedents and (ii) the only way to accommodate that behavior is by treating the meaning of counterfactuals as their context change potentials.

27 There are a number of equivalent ways of implementing things; in the text I opt for one that makes use of Lewis-style systems of spheres. A few small points: (i) even though officially what is counterfactually relevant can depend on the world at which we evaluate a counterfactual, since we are going to ignore iterated counterfactuals I'll often drop reference to the world; (ii) I'll treat the default situation as one where only the world of evaluation is counterfactually relevant (rather than a set including it); this is to make comparisons easy to see (for a set-up that doesn't go this way see Gillies 2007: §8); (iii) let's assume (as we have been sometimes doing already) that W is finite.

Conditionals presuppose that their antecedents are compatible with the relevant domain that the conditionals talk about. So counterfactuals presuppose that their antecedents are compatible with the counterfactual domain.

Entertainability Presupposition A counterfactual $(if\ p)(q)$ in context c presupposes that s_{π_c} contains some p-worlds.

Sometimes an assertion can be successful even if it has unmet presuppositions. That is because sometimes (ceteris paribus and within certain limits) we accommodate the missing presupposition and evaluate the assertion not with respect to the context as it was when the utterance was made but in the context changed to accommodate the missing presupposition.[28] And so it is with counterfactuals and their entertainability presuppositions. Asserting a counterfactual may change the conversational score when the presupposition is accommodated.

Definition 9 (Context Change via Accommodation)**.** Let c be any context and p any counterfactual antecedent. Then $c|p|$ is the posterior context that results form accommodating the entertainability of p into c where

$$\pi_{c|p|} = \{s \in \pi_c \colon\ s \cap [\![p]\!] \neq \emptyset\}$$

Notice that if s_{π_c} — the $\subseteq$-minimal domain in π_c — is compatible with p then $\pi_{c|p|} = \pi$. Otherwise $\pi_{c|p|}$ is the result of eliminating inner sets of worlds from π_c until we get one that has some p-worlds in it.

Putting the pieces together:

Definition 10 (Dynamic Strict Counterfactuals v.1.0)**.** Counterfactuals are dynamic strict conditionals that carry entertainability presuppositions:

$$[\![(if\ p)(q)]\!]^{c,w} = 1 \text{ iff } (s_{\pi_{c|p|}} \cap [\![p]\!]) \subseteq [\![q]\!]$$

This is equivalent to saying that the counterfactual is true iff the corresponding material conditional is $\Box_{s_{\pi_{c|p|}}}$-necessary.[29] So a counterfactual $(if\ p)(q)$ in a context c presupposes that p might have been the case and then asserts the strict conditional $\Box(p \supset q)$ with respect to the post-accommodation domain provided by c-changed-just-a-bit.

28 The classic references are Karttunen 1973, Stalnaker 1974, and Lewis 1979b.

29 There is room for refinement here if we want to address the (alleged) dynamics associated with *might*-counterfactuals (Gillies 2007: §7–8). That will take us too close to the bleeding edge for today.

Accommodation works by admitting as relevant the worlds as close as the nearest antecedent world.[30] Since we begin by default with the counterfactual domain containing just the world of evaluation, this means that the dynamic theory captures the variably strict theory as a special case when counterfactual discourse stretches only one conditional long.

Fact 11. Suppose context c delivers a system of spheres S_w. Then a counterfactual $(\textit{if } p)(q)$ is true at w in c according to the variably strict semantics iff it is true at w in c according to the dynamic strict conditional theory.

Things are different for sequences of counterfactuals. The dynamic theory treats counterfactuals as context dependent and context affecting. Context dependent because what the modal horizon is a function of context. Context affecting because what worlds are counterfactually relevant can be changed by a counterfactual's antecedent. It is this interplay that explains the asymmetry between (a)→(b) Sobel sequences like (12) and (14) and their (b)→(a) reverse Sobel sequences like (16) and (17).

Here's how. Suppose p is false at w but that the nearest p-worlds to w are q-worlds. For instance: suppose that Sophie did not go to the parade and Pedro was a conspicuous and reliably dancer at the parade. The default domain $\{w\}$ doesn't satisfy the counterfactual's presupposition. So we accommodate. It's possible for all the p-worlds in $s_{c|p|}$ to be q-worlds and so for $(\textit{if } p)(q)$ to be true.

Now we come to the second counterfactual in an (a)→(b) Sobel sequence. But the context is not the default initial context. It is $c|p|$. Still, it is possible that there are no r-worlds in $s_{\pi_{c|p|}}$. (Sophie is not overly tall and doesn't generally travel with stilts.) The presupposition of $(\textit{if } p \wedge r)(\neg q)$ isn't met so we accommodate. It's possible for all the $(p \wedge r)$-worlds in $s_{\pi_{c|p\wedge r|}}$ to be $\neg q$-worlds: the nearest worlds where Sophie gets stuck behind someone tall can all be worlds where she doesn't see Pedro dance.[31] So the counterfactual $(\textit{if } p \wedge r)(\neg q)$ can be true, too.

The (b)→(a) direction is different. If we first accommodate the antecedent of the (b) counterfactual $(\textit{if } p \wedge r)(\neg q)$ we go straightaway to context $c|p \wedge r|$. Assume that indeed all the $(p \wedge r)$-worlds in $s_{\pi_{c|(p\wedge r)|}}$ are $\neg q$-worlds so that $(\textit{if } p \wedge r)(\neg q)$ is true. There is now no room for the simple (a) conditional

30 We have put things in terms of systems of spheres. That was optional since there is an obvious back-and-forth between such a system $c(w)$ centered on w and an ordering $\leq_w$: $u \leq_w v$ iff u belongs to every sphere in $c(w)$ that v does (Lewis 1973).

31 Note that $c|p|(p \wedge r)| = c|(p \wedge r)|$.

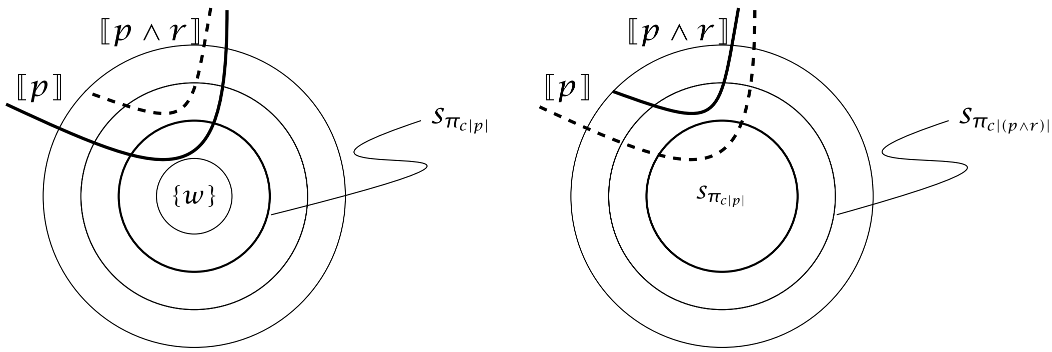

Figure 2 Accommodating p as a possibility (left) and $(p \wedge r)$ as a possibility (right)

to be true in the context $c|p \wedge r|$. That is because its presupposition — that p is possible relative to $c|(p \wedge r)|$ — is satisfied. Hence: no accommodation. But we just said that all the $(p \wedge r)$-worlds in $s_{\pi_{c|(p \wedge r)|}}$ are $\neg q$-worlds. And so it's false that all the all the p-worlds in $s_{\pi_{c|(p \wedge r)|}}$ are q-worlds. So the simple (a) counterfactual can't be true.

Members of Sobel sequences end up quantifying over different domains; members of reverse Sobel sequences end up quantifying over the same domain. No wonder there is an asymmetry. Figure 2 shows how this works graphically. For those who want to see this in slightly more formal terms, we need one more definition.

Definition 11. Let $C_0; \ldots; C_n$ be a sequence of counterfactuals and a_i be the antecedent of the ith counterfactual. Such a sequence is satisfiable iff there is a world w and a system of spheres $c(w)$ such that: (i) $[\![C_0]\!]^{c_0,w} = \cdots = [\![C_n]\!]^{c_n,w} = 1$, where (ii) $\pi^0_{c_0} = c(w)$ and (iii) $c_{i+1} = c_i|a_i|$ (for each $i > 0$).

This just captures what we have been assuming in working through the Sobel sequences: that interpretting counterfactuals changes the context and those changes can matter when it comes to seeing when a sequence is consistent or not.

Fact 12. Sobel sequences $(\textit{if } p)(q); (\textit{if } p \wedge r)(\neg q)$ are satisfiable according to the dynamic strict conditional theory. Reverse Sobel sequences $(\textit{if } p \wedge r)(\neg q); (\textit{if } p)(q)$ are not satisfiable.

The last word here has yet to be had. Counterfactual domains seem to get ever-larger. So one question: how do counterfactual domains get reset? Indeed there are examples (discussed in von Fintel 2001: §8 and Gillies 2007: §9) in which somehow or other the accommodation that a counterfactual's antecedent seems to trigger is promptly undone.

(18) If Sophie had gone to the parade and been stuck behind someone tall, she wouldn't have seen Pedro dance. Still, if Sophie had gone to the parade, she wouldn't have been stuck behind someone tall.

Some (for instance Moss (2012)) think the phenomenon tells decisively against the dynamic strict conditional theory. But there is room to move and the sort of thing that needs saying is clear: resetting happens, but it is not as smooth as expanding the counterfactual domain. There is evidence in favor of this, though the mechanism is poorly (i.e., not at all) understood. Some evidence that von Fintel discusses: modal and (certain) factual information makes the resetting smoother:

(19) If the USA threw its weapons into the sea tomorrow, there would be war. Well, if all the nuclear powers threw their weapons into the sea tomorrow, there would be peace. But of course, that would never happen. So, as things stand, if the USA threw its weapons into the sea tomorrow, there would be war.

(20) A: If John had been at the party, it would have been much more fun.
B: Well, if John had been at the party and had gotten into a fight with Perry, that wouldn't have been any fun at all.
A: Yes, but Perry wasn't there. So, if John had been at the party, he wouldn't have gotten into a fight with Perry.

Part of what makes this situation puzzling is that retreating to the variably strict theory does not seem like the right path. The reason has to do with negative polarity items (NPIs) like *any* and *ever* and when they are licensed. Some examples:

(21)
a. Every diner who has ever eaten there has liked it.
b. Every diner who ordered any dessert loved it.
c. *Some diner who has ever eaten there has liked it.
d. *Some diner who ate there has any reason to go back.

Quantificational claims like these combine a quantificational determiner Q with a restrictor A and a nuclear scope B, saying that Q-many of the As are B's. Some determiners (universal ones like *every*) are downward entailing in their first argument: $\textit{every}\,(A)(B) \models \textit{every}\,(A')(B)$ if $A' \subseteq A$. And some (like *some*) are not: $\textit{some}\,(A)(B) \not\models \textit{some}\,(A')(B)$ for some $A' \subseteq A$. (In fact *some* isn't downward entailing in either argument.) Instead *some* is upward entailing in its first argument. Ladusaw (1980) was the first to argue that NPIs are licensed only when they are in a downward entailing environment. That elegantly predicts the distribution in (21). It is in fact a very robust generalization.

Counterfactual antecedents can also contain NPIs. This (as argued in von Fintel 2001 and in von Fintel & Gillies 2015) is trouble-making for any retreat to a variably strict theory.

(22) a. If you had eaten any fruit, you would have had an apple.
b. If you had ever been to Idaho, you would have liked it.

The trouble is that the antecedent of a variably strict conditional is not a downward monotone environment. Given our set-up we can say something very stark.

Fact 13. A conditional $(\textit{if}\ \cdot)(\cdot)$ is downward entailing in its antecedent iff it supports antecedent strengthening.

Antecedent strengthening is of course just what the variably strict conditional is designed to do without. Whence it leaves unexplained why counterfactual antecedents license NPIs. The variably strict conditional theory gets this wrong for exactly the reason the strict conditional theory gets it right.

6 Indicative Conditionals and Collapse

We know that logical space is limited for a conditional that is bounded by strict implication (from Fact 5) and material implication (from below) and supports import/export. Material conditionals seem to be the only option. So if you are not satisfied with the horseshoe as a theory of ordinary indicative conditionals, you have choices to make. They are not easy choices.[32]

Another collapse argument:

32 Should we deny that indicatives are bound from above by strict implication? I know of no good reason to pursue this.

(23) Either the butler did it or the gardener did.
So: if it wasn't the butler, it was the gardener.

Patterns like this — the direct argument or more descriptively *or*-to-*if* — seem like entailments. But that makes for immediate trouble.

Fact 14. If $p \vee q \models (if\ \neg p)(q)$ then $p \supset q \models (if\ p)(q)$.

The proof is simple: suppose $p \supset q$ is true (at w in c). Then so is $\neg p \vee q$ and hence by *or*-to-*if* so is the indicative $(if\ p)(q)$. If indicatives are bound from below by material conditionals, then we have the stronger collapse conclusion: $(if\ p)(q) \dashv\models p \supset q$.

These seem like puzzles. Perhaps even puzzles-in-the-pejorative-sense (whatever that is). But the issue collapse arguments like these bring out is bigger than that. Indicative conditionals seem to be stronger than their corresponding material conditionals. They seem to express something more. But what this more is and how it gets expressed is not obvious. Collapse arguments like these are good ways of mapping logical space for how to think about this problem. But they also reveal that the going won't be easy (the assumptions they require are modest and super plausible).

So, one option is to deny the lower bound. If that is right then the truth of a counterexample $(p \wedge \neg q)$ isn't always sufficient for the falsity of the indicative $(if\ p)(q)$. And so, if indicatives do not entail their corresponding material conditionals, it is possible for $(if\ p)(q)$ and p to both be true, but for q to be false. It's not enough to just deny an assumption: what we need is a reason for thinking we should drop it. So we need reasons to think $(p \wedge \neg q)$ isn't always sufficient for the falsity of the indicative $(if\ p)(q)$. And there are (some say) concrete examples to be found. McGee's (1985) example:

(24) If a Republican won, then if Reagan didn't win Anderson did.

It seems possible to be in favor of a right-nested indicative like (24), and to be in favor of its antecedent, without being particularly attracted to the embedded conditional considered on its own. That doesn't yet fit the bill for what needs doing, but it is a start.[33] It is safe to say that this has not turned out to be the favored route.

Another option: perhaps indicative conditionals do not support import/export. As we have already seen, variably strict conditionals do not. So if indicatives are variably strict, then they won't. Stalnaker (1975) — rightly, I think — takes counterfactuals and indicatives to share the same core semantics. Since there are good reasons for the variably strict picture of counterfactuals, there are good reasons for that picture here, too. The reasons (as we'll see) have little to do with import/export.

Indicative and counterfactuals are different: witness Adams pairs like (3a) and (3b). The idea is that they nevertheless share the common semantic core of variably strictness. What sets indicatives apart is a pragmatic constraint on their proper use: indicative conditionals say something about the not-yet-ruled out worlds in a context.

Pragmatic Constraint If p is compatible with the context c, then the $\leq_w$-minimal p-world(s) are compatible with c.

The constraint can be suspended — that, arguably, is what the distinctive tense/aspect marking of "subjunctive" conditionals does.

Strict conditionals are stronger than material conditionals, so we know that (under plausible assumptions about entailment) *or*-to-*if* inferences like (23) can't be entailments.[34]

Fact 15. If $(\textit{if}\ \cdot)(\cdot)$ is a variably strict conditional then $p \vee q \not\models (\textit{if}\ \neg p)(q)$.

Why then do instances of this pattern like (25) seem like entailments?

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33 We can't do justice to the literature surrounding McGee-style counterexamples (but see the references in Bennett 2003 and the more recent discussions in Gillies 2004, Weatherson 2009, and Huitink 2012). Along these lines: there has been some resurgent interest lately in generating "counterexamples" to various principles for conditionals. (See, for instance, Kolodny & MacFarlane 2010.) I am of the opinion that this is largely a mischaracterization of the issues. For example: there is no way to say what counts as "modus ponens" without saying what counts as "entailment". And where some might see modus ponens failures I'm more inclined to see a poor choice for a story about entailment. A less cryptic version of the point can be found in Gillies 2009.

34 Counterexample: suppose there are just two c-relevant worlds, w and v, where w is a $(\neg p \wedge q)$-world and v is a $(p \wedge \neg q)$-world. Then $p \supset q$ is true at w in c. Since p is compatible with c, the $\leq_w$-minimal p-world must also be compatible with c: so it's v. But since v is a $\neg q$-world, then $(\textit{if}\ p)(q)$ isn't true at w.

(25) Either the butler did it or the gardener did it.
So: if the butler didn't do it, then the gardener did it.

Stalnaker's answer: because there is a nearby pragmatic property — reasonable inference — that they do have.

Definition 12 (Reasonable Inference)**.** Suppose p is successfully asserted (at w in c) and c' is the resulting posterior context. p, *so*: q is a reasonable inference iff q is accepted in c'.

Suppose that in order for a disjunction $p \vee q$ to be appropriately asserted in a context c that c has to have some $(p \wedge \neg q)$-worlds compatible with it and also some $(\neg p \wedge q)$-worlds compatible with it.[35] To be accepted in a context a sentence must be true at every world compatible with it. So if a disjunction is successfully asserted in c then $\neg(p \wedge q)$ is accepted in c'. Then anytime $p \vee q$ is appropriately asserted we will end up in a context in which the indicative $(\mathit{if}\ \neg p)(q)$ is accepted.[36]

Fact 16. If $(\mathit{if}\ \cdot)(\cdot)$ is variably strict and obeys the pragmatic constraint on indicatives then *or*-to-*if* is a reasonable inference.

This is an elegant solution. A crucial part of it is how it uses an independent and plausible pragmatic fact about disjunctions — that they are felicitous only when either conjunct might be true without the other — to get where we need to go. Therein lies the elegance. But there is no similar thing to be said about either right-nested conditionals like $(\mathit{if}\ p)((\mathit{if}\ q)(r))$ or their exported counterparts like $(\mathit{if}\ p \wedge q)(r)$. So the strategy here leaves unexplained why import/export seems valid.

Collapse arguments rely (sometimes tacitly, sometimes not) on the assumption that indicative conditionals fit in with the rest of our linguistic tools in being vehicles that can be either true or false and have the same sort of semantic values that non-conditional language has. For that reason collapse arguments can be taken to be arguments for the thesis that indicative

35 This runs into some trouble if either p or q is necessary. That's likely a feature of the possible worlds framework as much as a bug of the quasi-gricean idea here about disjunction, so let's set it aside.

36 Argument: suppose $p \vee q$ is appropriately asserted at w in c. So c has both $(p \wedge \neg q)$-worlds and $(\neg p \wedge q)$-worlds compatible with it. Let c' be the posterior context; it has only $(p \wedge \neg q)$-worlds compatible with it. Let v be any world in c'. The indicative $(\mathit{if}\ \neg p)(q)$ requires that the nearest $\neg p$-world(s) to v be compatible with c'. So they must be q-worlds.

conditionals do not, after all, have truth-values or express normal semantic values. This is sometimes called the No Truth Value (NTV) theory.[37] The idea is that indicatives do not represent or report conditional information at all. Instead they are vehicles for expressing states of mind, in particular that the speaker has sufficiently high credence in the consequent given the antecedent. That is: they aren't things we assert, but things we use to make conditional assertions.[38]

There is a lot to be said for tying the appropriate use of conditionals to conditional assertion.[39] But NTV views are also hard to reconcile with a uniform picture of semantics. And, indeed, they are hard to square with why it is that conditionals mix with other, non-conditional, bits of language: they can be negated, they embed modal vocabulary, and go in for quite a bit of nesting. Take the modals.

(26) a. If Red isn't in the box, Blue must be.
b. If Red isn't in the box, Blue might be.

Hypothesis: epistemic modality and indicative conditionals are tightly linked. Whatever kind of semantic-plus-pragmatic bundle we opt for in one case we should opt for the same sort of bundle in the other case. The dilemma in the offing here is that NTV either extends the thesis to the modals or not. If so, then since *must* and *might* aren't for asserting on their own, embedding them can't be something we do to conditionally assert them. So (26) are prima facie out. If not, then the attractive hypothesis linking modals and conditionals has to go.[40]

Even so we may be driven in the NTV direction all the same. One reason (we won't really be able to do justice to it here): Lewis (1976) showed that (on pain of triviality) there is no two-place propositional operator such that the probability of the resulting proposition being true when given arguments p and q always equals $\Pr(q|p)$. Hence:

37 See Adams 1975, Gibbard 1981, Bradley 2000, Edgington 2009, Bennett 2003.

38 This is different from the versions of the horseshoe theory in Lewis 1976 and Jackson 1991 and Lewis 1986 according to which indicatives have the truth conditions of material conditionals but which, owing to some pragmatic wrangling, have high (enough) conditional credence as assertability conditions.

39 Stalnaker 2005 tries to narrow the gap between a conditional propositions picture and a conditional assertion picture.

40 There are things that are sometimes said on behalf of the NTV view here (see, for instance, Bennett 2003) but I don't find them compelling. A more thorough discussion can be found in Gillies 2012.

Fact 17. If $(\mathit{if}\ p)(q)$ expresses a semantic value that determines a truth-value then it is not in general true that $\Pr((\mathit{if}\ p)(q)) = \Pr(q|p)$.

Insofar as we want our attitudes toward indicatives — the degree to which we believe them or want to assert them or bet on them or whatever — to track the corresponding conditional probabilities (and we do) that is an argument against assigning them truth-values.[41] But the argument can be resisted. As we will see, some theories (notably the "restrictor" theory developed by Kratzer, but also some others) seem to take triviality in stride.[42]

Another reason, due to Edgington (2009: §2): if indicatives have truth conditions then those truth conditions are either truth-functional or non-truth-functional. Neither seems right for indicatives. Consider non-truth-functional truth conditions. (Truth-functional truth conditions are a dead end because the horseshoe is the only option.) Lack of truth-functonality requires variability. That is: the truth values of p and q at w (in c) don't fix the truth value of $(\mathit{if}\ p)(q)$ at w (in c). So if p is false at w and q is true at w then sometimes $(\mathit{if}\ p)(q)$ is true at w and sometimes it isn't. But indicatives seem to validate *or*-to-*if* and this requires uniformity. That is: if all the information you have is that $\neg p \vee q$ then that is always sufficient for $(\mathit{if}\ p)(q)$. Edgington says variability and uniformity are at odds and so no non-truth-functional theory can't get this right. Conclusion: no way of assigning truth conditions to indicatives can be right.

This is a compelling argument but it isn't airtight since there is room for a non-truth-functional theory to embrace both variability and uniformity. It is possible that the variability required by non-truth-functionality be variability in truth value at a given world between contexts. And it is possible that the uniformity required by *or*-to-*if* be uniformity across worlds compatible

41 The basic result has been extended and fortified many times over: see, for instance, Lewis 1986, Gärdenfors 1982, Hájek 1989, 1994 and in the qualitative domain Gärdenfors 1986, Segerberg 1989.

42 For those in the know or those returning to this section after we discuss the restrictor theory in Section 7: the idea there is that when indicatives and probabilistic hedges mix, the correct logical form isn't one of a conditional scoping under (or over) a probability operator. Rather it's a dyadic probability operator whose first argument is the "antecedent" and whose nuclear scope is the "consequent". This idea is advanced in Kratzer 1986, 2012 and resisted in various ways in Rothschild 2012 and von Fintel & Gillies 2014.

with a given context. The master argument overlooks the possibility of such theories of indicatives.[43]

7 Antecedents as Restrictors

So far we have only been worrying about what various sorts of conditionals mean. There is an orthogonal issue: how are those meanings achieved by the *if*-of-English? We have assumed a straightforward answer: that the *if*-of-English is adequately represented in our regimented language by a binary connective $(\mathit{if}\ \cdot)(\cdot)$ and that this is assigned a uniform semantic value. But this is an assumption and can be challenged. In philosophy *can be challenged* often enough implies *has been challenged*. This is such a case.

Lewis (1975) argued that *if*s — apparently conditional expressions — in certain quantificational environments do not express any conditional thing. Example:

(27) $\left\{\begin{array}{r}\text{Always}\\ \text{Sometimes}\\ \text{Seldom}\end{array}\right\}$ if a farmer owns a donkey, he beats it.

What single thing could *if* mean in each of these and could that single thing be a conditional operator rightly so-called? The *if* might plausibly contribute some iffy meaning — for instance a strict conditional of some sort — when the quantifier is a universal like *always*. But conjunction is a better candidate if it's an existential like *sometimes*. Neither looks good for *seldom*.

In asking the question we assumed that we have two operators, a quantifier $\mathcal{Q}$ and a conditional $(\mathit{if}\ \cdot)(\cdot)$ and that the relevant structure of (27) sorts the scopes out this way:

(28) $\mathcal{Q}\,(\mathit{if}\ p)(q)$

That gets us into trouble so Lewis's (1975) conclusion is that sentences like those in (27) are not instances of a conditional operator plus an adverb of quantification.[44] We don't, appearances aside, have the two operators $\mathcal{Q}$ and $(\mathit{if}\ \cdot)(\cdot)$. Instead, he said, the *if*s in environments like these are a

43 Dynamic or so-called "context-shifty" accounts, as we'll see in Section 8, occupy just this spot in logical space. The master argument overlooks this possibility. The connection to Edgington's master argument is discussed in Gillies 2009: §4.

44 Saying that the scoping is $(\mathit{if}\ p)(\mathcal{Q}q)$ fares no better.

non-connective whose only job is to mark an argument slot for the adverb of quantification.[45]

The trouble-making feature is that *if*s under adverbs of quantification express restricted quantificational claims. In (27) those are claims about donkey-owning-farmer situations: that they are always/sometimes/seldom farmer-beating-donkey situations. There seems to be no way for the *if*s to both contribute a uniform conditional meaning and do this restricting. Since these *if*s restrict, they aren't iffy.

Kratzer's idea is that this holds not just for *if*s under adverbs of quantification but for all *if*s.[46] The picture is that *if* doesn't contribute a meaning, much less a conditional meaning, to the sentences in which it occurs:

> The history of the conditional is the history of a syntactic mistake. There is no two-place "if. . . then" connective in the logical forms for natural languages. "If"-clauses are devices for restricting the domains of various operators. (Kratzer 1986: p.11)

The thing all *if*s do is restrict operators (quantifiers, modals). So they aren't iffy. (What about bare conditionals, conditionals which appear not to have a nearby operator? Since the hypothesis is that *if*s are devices for restricting operators, there must be one. So we posit a covert, universal operator.)

Definition 13 (Restrictor Analysis)**.** Conditionals are restricted operators with logical forms like this:

Quantifier/Operator + *if*-clause + consequent clause

$$\textsc{operator}(p)(q)$$

45 Obligatory quote:

> The *if* of our restrictive *if*-clauses should not be regarded as a sentential connective. It has no meaning apart from the adverb it restricts. The *if* in *always if* . . . , . . . , *sometimes if* . . . , . . . , and the rest is on a par with the non-connective *and* in *between* . . . *and* . . . , with the non-connective *or* in *whether* . . . *or* . . . , or with the non-connective *if* in *the probability that* . . . *if* . . . . It serves merely to mark an argument-place in a polyadic construction. (Lewis 1975)

46 Developed (with some variations) in, e.g., Kratzer 1981a, 1986, 1991, 2012.

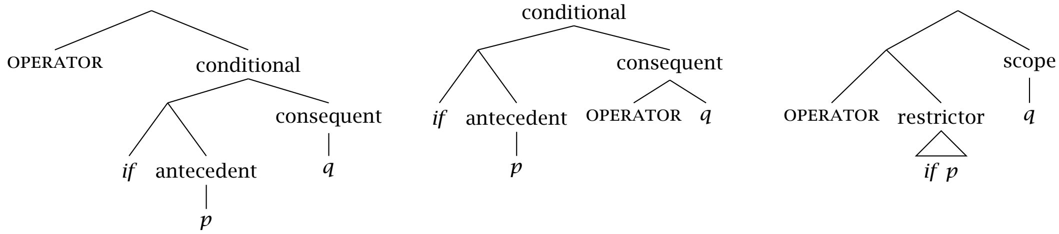

Figure 3 Operators + *if*: Conditional vs. Restrictor

Two things to notice. First: the binary connective $(if\ \cdot)(\cdot)$ makes no appearance. The action is all about the operator. Second: for just that reason we won't have a story about "indicative conditionals" or "counterfactual conditionals". Instead, as we'll see, we get a story by saying something about the operators that the relevant *if*-clauses restrict. That's why this is less an issue about <u>what</u> various conditionals mean and more an issue about <u>how</u> natural language sentences manage to carry those meanings.

Figure 3 sketches the differences in structure. The left and center trees illustrate the structure where an operator and a conditional connective mix: the left is the case where the operator outscopes the conditional with antecedent p and consequent q and the center is the case of a conditional with antecedent p and complex consequent OPERATOR q. The rightmost tree shows the structure for the restrictor analysis. Here the "antecedent" is the restrictor for OPERATOR and the "consequent" q is the nuclear scope. (We'll now focus on the case where the operators involved are modals (as opposed to quantifiers).)

Take an ordinary counterfactual like (29):

(29) If Alex had been there, she would have had fun.

So far treating the (counterfactual) *if*-clause as a restrictor doesn't do much because we haven't yet said what the modal *would have* means.

In principle we could adapt just about any theory about what such conditionals mean to what such modals mean. Here we will (with some simplifications) follow Kratzer and give an analysis that makes the same predictions as the variably strict story. Here is the idea: modals quantify over a domain, saying that all or some of the best worlds in it (and in which some additional thing (provided by a restricting *if*-clause if there is one) is true) satisfy the prejacent (the scope). Context supplies the domain (the modal base at a world) and the notion of bestness (induced by the ordering source at a world).

Definition 14 (Modal Base, Ordering Source)**.** A conversational background is a function from worlds to sets of propositions. A modal base B is a conversational background. Similarly, an ordering source O is a conversational background. The ideal worlds at w relative to B and O is the set of worlds that are $\leq_w^O$-minimal in $\bigcap B(w)$ (where $\leq_w^O$ is the ordering induced by the set of propositions $O(w)$ used in Fact 10).

Contexts determine both a modal base and an ordering source. These are the same type of object, but they play different roles. Modal bases simply form domains. We aren't making use of their additional structure here, so we just take the image of a base B_w to get the worlds compatible with all the propositions in it. The extra structure in ordering sources — the extra structure over and above a set of worlds — is used. We use it to induce an ordering. The two pieces together get us the ideals.[47]

Context has the job of selecting values for the modal base and ordering source.[48] Once it does, the modal simply contributes (restricted) quantificational oomph.

Definition 15 (Modals)**.** Suppose B is the modal base and O the ordering source determined by context c. Then:

i. $[\![$*must* / *would* $(p)(q)]\!]^{c,w} = 1$ iff for every ideal v at w wrt $B \cup \{[\![p]\!]\}$ and O: $[\![q]\!]^{c,v} = 1$.

ii. $[\![$*might* / *might have* $(p)(q)]\!]^{c,w} = 1$ iff for some ideal v at w wrt $B \cup \{[\![p]\!]\}$ and O: $[\![q]\!]^{c,v} = 1$.

The restrictor p simply augments the modal base. And so, together with Definition 13, that's all that *if*-clauses do.[49]

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47 As before, and for the same reasons, the Limit Assumption (or finiteness, take your pick) is in play here.

48 That's no different from the variably strict story where contexts were asked (somewhat implicitly) to supply an ordering of relative similarity $\leq_w$ or a set of premises P_w for each world w.

49 Two questions. What if there is no restrictor (that is, if the modal claim is something like *Alex must be there*)? Then take the first argument to be your favorite tautology. What if there is no modal at all (that is, if the conditional is a bare conditional like *If Alex is at the party,*

We have barely constrained what sorts of functions the backgrounds can be. Adding constraints gets us different possibilities for the flavor of the modal involved and the sort (and strength) of the conditional. Some possibilities for a conversational background f:

- f is realistic iff for every w: $w \in \bigcap f(w)$.
- f is totally realistic iff for every w: $\bigcap f(w) = \{w\}$.

Backgrounds, like accessibility relations and accessibility functions, can also characterize sorts of contextually relevant information:

- f is an epistemic background iff (for every w) $f(w)$ is the set of propositions known to the c-relevant agents at w.
- f is an stereotypical background iff (for every w) $f(w)$ is the set of propositions which normally hold at w.

And so on.

This predicts for a wide range of possible contextual resolutions for conditionals.

Fact 18. Material conditionals correspond to the special case where B is totally realistic and O is empty. Strict implication corresponds to the special case where both B is empty and O is empty.

To get a feel for this, let's walk through the material conditional case. Let B be a totally realistic background and O be empty. Then a conditional *if* p, (*must*) q—that is, *must* $(p)(q)$—at w says that the best worlds in $\bigcap B(w)$ that are p-worlds are all q-worlds. But $\bigcap B(w) = \{w\}$ and the induced-ordering treats all worlds alike. So whether the conditional is true is down to w alone. Suppose p is true at w. Then the conditional is true iff w is a q-world. Suppose p is false at w. Then the conditional is vacuously true at w since there are no p-worlds in $\bigcap B(w)$. These are just the truth-conditions of the material conditional.

The empirical hypothesis is that counterfactuals are conditionals (which is to say "conditionals") interpreted against an empty modal base and a totally realistic ordering source. This is backed-up by the following:

she is outside)? Since *if*s restrict, there must be one. So posit a covert necessity operator (a covert *must*).

Fact 19. Let B be the empty modal base and O be the c-relevant totally realistic ordering source. Now take the premise set P to be such that for every w: $P_w = O(w)$ Then: $[\![\mathit{would}\,(p)(q)]\!]^{c,w} = 1$ according to the restrictor analysis (Definition 15) iff $[\![(\mathit{if}\ p)(q)]\!]^{c,w} = 1$ according to the premise set analysis (Definition 7).

So premise sets correspond to special ordering sources and the analyses use them in the same way. Hence the (promised, and now delivered) equivalence.

All of this carries over to the case of indicatives. There the relevant modals are epistemic, taking an epistemic modal base and an empty ordering source. Doing things that way, as Kratzer (1986) argues, relieves some of the pressure in the collapse arguments we saw in Section 3 and Section 6. I want to mention just two examples for now.

The first: import/export. The trouble was that we couldn't, without collapse to the material conditional, satisfy both this and have the truth of a counterexample $(p \wedge \neg q)$ always suffice for the falsity if the corresponding indicative $(\mathit{if}\ p)(q)$. (Or, in this case, the corresponding modal $\mathit{must}\,(p)(q)$.) Epistemic modal bases are realistic (since you can't know false things), so it would seem that we can't have import/export. But there is room for maneuvering for the restrictor view.

Fact 20. $\mathit{must}\,(p)(\mathit{must}\,(q)(r)) \dashv\vDash \mathit{must}\,(p \wedge q)(r)$

How is this possible? Kratzer says that Gibbard's proof "does not threaten" this analysis, but doesn't really say more than that. The answer lies in the fact that conditionals ("conditionals") express different things when they occur on their own compared to when they occur embedded.[50]

The second: it is hard to see how the restricting behavior gets done if *if* expresses a bona fide conditional. This is part of what is going on in Lewis-style triviality arguments. But there is an easy way out.

(30) a. If the bet is on odd, it's probably a loser.
$\mathit{probably}\,(p)(q)$
b. If the coin is fair, then the probability of heads is $\frac{1}{2}$.
$\frac{1}{2}$-$\mathit{probable}\,(p)(q)$

Filling in very off-the-shelf semantics for the probability modals here immediately predicts that (30a) is true iff the corresponding (c-relevant) conditional

50 See Gillies 2009, Khoo 2013.

probability is high enough and that (30b) is true iff the relevant (c-relevant) conditional probability is .5. Triviality doesn't threaten because there is no separate conditional proposition that the probability-modal takes scope over.[51]

Again, in principle the *if*s-as-restrictors picture is compatible with just about any story of the relevant modals and so can be made to come out equivalent to just about any story of conditionals. To reiterate: that fact goes in the pro column since one of the achievements here is to separate out questions about how conditional meanings get expressed from questions about what those conditional meanings are.

8 Dynamics and Indicative Conditionals

The dynamics of counterfactual domains highlights ways in which conditionals can be seen as externally dynamic: interpreting one sentence changes an essential component of conversational score that is relevant for the interpretation of a later sentence. Conditionals can also be thought of as internally dynamic: interpretation of one part of a sentence changes an essential component of conversational score that is relevant for the interpretation of a later bit of that same sentence.[52]

Let's explore that by sketching a dynamic semantics of indicative conditionals that is inspired by the Ramsey test. In fact, we will see two implementations of that theory: one implementation in a dynamic framework and one using the (seemingly) more familiar context-plus-index-delivers-truth-values set-up we've mostly been assuming throughout. The first implementation is in Gillies 2004 and the second in Gillies 2009, 2010.[53] Taking indicatives to be dynamic (strict) conditionals offers a new perspective on the various collapse arguments, Edgington's "master argument" against truth-conditions,

51 This is a very attractive way out, one hinted at (but somehow not pursued) by Lewis himself when he calls the *if* in *the conditional probability of . . . if . . .* a non-connective. It has, however, recently been challenged (Rothschild 2012, von Fintel & Gillies 2014).

52 For those familiar with dynamic predicate logic (Groenendijk & Stokhof 1991): the existential quantifier is externally dynamic since its (semantic) binding power outruns its (syntactic) scope; conjunction, which is relational composition, is internally dynamic.

53 The original idea was inspired (in roughly equal parts) by Ramsey's footnote, the data semantic treatment of indicatives in Part III of Veltman 1985, and the simple update semantics for epistemic modals in Veltman 1996. (The inspiration and divergences, I trust, will be clear.) There is a point to having both implementations available, but we will get to that.

and a modal parallel to Lewis's argument about *if*s under adverbs of quantification.[54]

We have been assuming throughout that semantic values are sets of points of evaluation: they are the kind of thing that, when supplied a world, deliver a truth-value (that's what Figure 1 illustrates). Nothing prevents us from using this set-up to understand more complicated object languages than we have so far. For instance, we might have an object language that includes recipes and programs.[55] A regular (modal) sentence has a set of worlds as its semantic value because what it says is that things are a certain way. Programs aren't like that. They instead say what they do to ways the world might be: execute it in a world like this, such-and-such results; execute it in a world like that, thus-and-so results. So the denotation of a program is a relation, a set of ordered pairs of worlds (or "states" or whatever the points of evaluation are): the set of ordered pairs such that executing the program in the first member of the pair (possibly) terminates in the second member of the pair. Dynamic semantic theories can be thought of as theories that say all sentences are like that: they are recipes or programs for changing the context or conversational score.[56]

So much for broad brushstrokes. In order to make this concrete we need to decide what aspects of conversational score we care about or what sort of information we want represented in a context. (We will do this in an update semantic framework (a là Veltman 1996) and follow custom by sometimes calling such bodies of information information states.) For us this decision is easy: sets of not-yet-ruled out possibilities.

Definition 16 (Contexts, Information States)**.** A context or information state s is a subset of the set W of possible worlds. I is the set of all such s's.

The limit cases of contexts I (the state of total ignorance) and $\emptyset$ (the absurd state) will have roles to play.

54 There are further refinements, developments, and purposes the dynamic perspective on conditionals can be put to that we won't be able to do justice here. See, for instance, Starr 2014, Willer 2014.

55 That's what propositional dynamic logic, a particular modal logic, is. A good reference: Harel et al. 2000.

56 The theories I have most in mind here as exemplars of (classic) dynamic semantic theories are those in Groenendijk & Stokhof 1991 and Veltman 1996. Besides the analogy to the semantics of programming languages (and the empirical coverage they offer), there are other inspirations for the early dynamic turn, especially Karttunen 1973, Stalnaker 1974, and Lewis 1979b.

Sentences of our intermediate formal language (that is, propositional logic plus $(if\ \cdot)(\cdot)$) have the same type of semantic value as programs. They are context change potentials, relations between information states (actually: functions from states to states).[57]

Definition 17 (Context Change Potentials (CCPs))**.** Let a be any atomic formula, p, q be any formulas and s be any information state. Then the semantic value function $[\cdot]$ is defined as follows:

i. $s[a] = \{w \in s\colon\ w(a) = 1\}$

ii. $s[\neg p] = s \setminus s[p]$

iii. $s[p \wedge q] = s[p][q]$

iv. $s[(if\ p)(q)] = \{w \in s\colon\ s[p][q] = s[p]\}$

Glossing the machinery for the first three clauses: (i) the change induced by a successful assertion of an atomic sentence is to remove possibilities in which the atom isn't true; (ii) negation corresponds to complementation; and (iii) conjunction is functional composition (the state $s[p]$ is the input for the function $[q]$). All of these represent programs whose purpose is to change the conversational score. The indicative conditional is a different sort of program. It is a test: it checks to see if the state $s[p]$ is that same as the state $s[p][q]$. If so, the result is the original s; otherwise, the test fails and the result is $\emptyset$.

This is represented in Figure 4 where CCPs are transitions between states. Updating s with $[(if\ p)(q)]$ loops to s iff updating $s[p]$ with $[q]$ loops to $s[p]$. That is the situation in the left diagram. On the right: $s[(if\ p)(q)] = \emptyset$ because updating $s[p]$ with $[q]$ adds something new, carrying us to new state t.

This represents a conservative extension of the ordinary semantics of propositional logic.

Fact 21. Let p be any if-free formula and s any state. Then: (i) $W[p] = [\![p]\!]$ and (ii) $s[p] = s \cap [\![p]\!]$.

The indicative, though, makes a difference. The way this is often demonstrated is to note that the update function $[\cdot]$ is not, in general, distributive in the sense that it doesn't respect arbitrary unions of its arguments.

57 The notation in Definition 17 is post-fix: s is the argument to the function $[p]$.

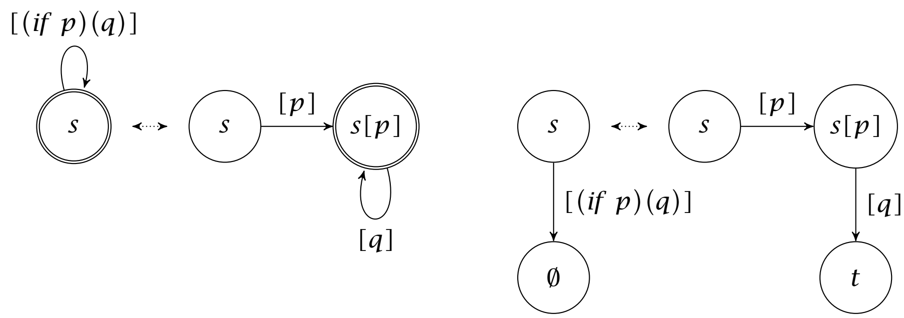

Figure 4 Conditional updates: Accepted (left) and not (right)

Fact 22. It is not in general the case that $s[p] = \bigcup_{w \in s} \{w\}[p]$.

Negated conditionals are a counterexample: updating a state $\{w, v\}$ where $w(a) = w(b) = v(a) = 1$ and $v(b) = 0$ with $\neg\,(\textit{if } a)(b)$ returns $\{w, v\}$ but updating the singletons $\{w\}$ and $\{v\}$ and collecting the results returns just $\{w\}$.

So indicatives are strict conditionals that invite tests. But when are they true? And what about entailment? Since we are in a richer logical space, we have options. These are reasonable choices: truth is a fixed-point, entailment is so-called update-to-test entailment.[58]

Definition 18 (Truth, Entailment)**.** For any sentences p, p_i, q and any state s:

i. p is true in s (or s supports p) iff $s[p] = s$.

ii. $p_1, \ldots, p_n \models q$ iff for any s: q is true in $s[p_1] \ldots [p_n]$.

Together with Fact 21 it follows that (for *if*-free p) $s \models p$ iff $s \subseteq [\![p]\!]$. This picture of indicatives is Ramseyan in that the truth of an indicative in a state rides on whether a derived or subordinate sub-state supports the consequent.

Fact 23. An indicative $(\textit{if } p)(q)$ is true in s iff q is true in $s[p]$.

This too can be seen in Figure 4 where double-circled nodes represent supporting states.

58 Much of what we know about the landscape here — what options there are, what the properties of those options are — is due to Veltman (1996), van Benthem (1996), and van der Does et al. (1997).

Collapse arguments highlight the puzzle indicatives pose: they seem to say more than material conditionals but there are no good candidates for what that more is (or worse: it seems impossible for them to say more). The dynamic turn opens up possibilities.

Fact 24. The dynamic conditional together with update-to-test entailment supports modus ponens and import/export. Moreover, if p implies q then $(\textit{if}\ p)(q)$ is true in any state and the $(\textit{if}\ \cdot)(\cdot)$ is stronger than the corresponding material conditional.

This was a package that collapse arguments led us to believe was not possible.

How does this square with the fact that modus ponens plus import/export lead to collapse (Fact 5)? The frameworks are different and so comparisons non-obvious. Let's focus on the conditional-only fragment of the language and (following Gillies 2009, 2010) re-tell things in the context-plus-index-delivers-truth-values set-up.

As in Section 3, contexts deliver sets of possibilities relevant for indicatives. We will again take contexts themselves to be such functions. Here we choose two properties such functions must have.

Definition 19 (Well-behaved Contexts)**.** c is well-behaved iff for any w:

i. c is reflexive: $w \in c(w)$;

ii. c is euclidean: if $v \in c(w)$ then $c(w) \subseteq c(v)$.

The first means that the facts at w are always relevant to the truth of indicative at w. The second means that what is relevant at w is settled. Reflexive and euclidean functions are closed: if $v \in c(w)$ then $c(w) = c(v)$.

The theory here will be a strict conditional theory (different, as we'll see, from the one in Definition 2). So it will say that $(\textit{if}\ p)(q)$ is true at w in c just in case the p-worlds in $c(w)$ are all worlds at which q is true. But true at what context? The Ramseyan subordinate or derived context — write it $c + p$ — got by hypothetically adding the information of the antecedent to c.

Definition 20 (Shifty Strict Conditional)**.** For any p, q in a well-behaved context c:

$$[\![(\textit{if}\ p)(q)]\!]^{c,w} = 1 \text{ iff if } v \in c(w) \text{ and } [\![p]\!]^{c,v} = 1 \text{ then } [\![q]\!]^{c+p,v} = 1$$

where $c + p = \lambda w.c(w) \cap [\![p]\!]^{c}$.

This replicates the dynamic strict conditional.[59] And so there is no collapse. Notice that this puts conditional antecedents to work twice over. They restrict the domain throughout which we check for the consequent's truth. Plain (non-shifty) strict conditionals do this, too. But here antecedents also contribute to the context that is relevant to the interpretation of those consequents. This is the internal dynamics we saw earlier and sets this apart. The shifting is essential. One way to see that: strict conditionals with no shifting can't validate import/export. Another way: earlier we saw that Edgington's master argument against truth-conditions seemed to pit variability against uniformity but that the argument overlooked the possibility that the variability is variability in truth-value at a world across contexts and the uniformity is uniformity across worlds within a context. The context-shifting delivers exactly that. And a third way: we saw that from the point of view of correspondence theory, strict conditionals that support import/export have peculiar selection functions (Fact 4). Not so here: with a context-shifty strict conditional the odd property of "shift coreflexivity" that characterizes one direction of import/export gets replaced by the much more tame property of transitivity.[60]

Let's wrap up this brief foray into the shifty/dynamic strict conditional by looking at one more way the dual-roles assigned to antecedents makes a difference. The problem that (27) exposes for *if*s that mingle with adverbs of quantification have parallels for *if*s that mingle with (epistemic) modals.

(31) a. If Red isn't in the box, Blue must be.
b. If Blue isn't in the box, Red must be.
c. If there is a marble in the box, it might be Red.

Assume that *must* here expresses a strong necessity epistemic modal $\Box$ over the set of relevant worlds.[61] And assume that *might* is its dual. Then it is surprisingly difficult to say that *if* contributes the same iffy meaning to all the examples in (31).

Here's part of the trouble.

(32) Red might be in the box and Blue might be in the box.

59 Officially we need to define an entailment relation (Gillies 2009: §6). The simple case: $p_1, p_2 \models q$ iff if $[\![p_1]\!]^{c,w} = 1$ and $[\![p_2]\!]^{c+p_1,w} = 1$ then $[\![q]\!]^{((c+p_1)+p_2),w} = 1$.

60 See Kaufmann & Kaufmann 2015.

61 Each part of this assumption is defensible — that *must* is strong (von Fintel & Gillies 2010) and that it quantifies over the *if*-relevant worlds (Gillies 2010: §6).

This is perfectly compatible with (31a) and (31b). For instance: suppose I know that one and only one of my marbles — Red, Blue — is in the box. But for nearly every p and q the conditionals $(if\ \neg p)(\Box q)$ and $(if\ \neg q)(\Box p)$ aren't jointly compatible with $\Diamond p \wedge \Diamond q$. The restrictor theory has no trouble here. [62]

The broader issue here that parallels what Lewis saw with adverbial quantifiers is that *if*-clauses restrict these modals. But it is hard to see how that restricting can get done by a single conditional operator: one sort of operator might seem reasonable if the operator is a universal like *must* but that same operator looks pretty bad for an existential comingling modal like *might*.[63] This is an issue that shifty/dynamic conditionals handle with ease.

Fact 25. Given Definition 19 and Definition 20:

i. $(if\ \neg p)(\Box q)$, $(if\ \neg q)(\Box p)$ are jointly compatible with $\Diamond p \wedge \Diamond q$

ii. $(if\ p)(\Diamond q) \dashv\models \Diamond(p \wedge q)$

These are the same predictions that the restrictor analysis achieves but here *if* still makes a uniform contribution. That is because of the Ramseyan core: the antecedent contributes to the subordinate or derived context that the embedded modal is sensitive to.

62 The reply, invariably, is that we should wide-scope the modals. This turns out to force a choice between giving up a link between *if*s and *must*s:

(i) If Alex is here, Billy is here ≈ If Alex is here, Billy must be here.

and giving up a link between *if*s and *might*s:

(ii) If Alex is here, Billy might be here ≈ It might be that Alex and Billy are here.

The restrictor theory doesn't have to give up either. This is discussed in Gillies 2010: §6.

63 Lewis admitted that the restricting was after all possible with a "far-fetched" interpretation of the conditional — the one defended in Belnap 1970 that says that $(if\ p)(q)$ is true at w if the confirming instance is true, false if the counterexample is true, and undefined if p is either false or undefined at w. The idea has been revived and extended to be fit with modals (von Fintel 2007, Huitink 2009). It is problematic, though, since it seems to be saddled with the prediction that indicatives presuppose (entail, actually) their antecedents. That seems a distance from what conditional information is all about. (See also the discussion of this in von Fintel 2012: §5.2.)

9 Further Reading

No survey of conditionals can be both complete and digestible. This one aimed at the latter property at the expense of the former property and so strategic choices dictated that things which deserve more attention did not get their due. Other surveys — for instance, Bennett 2003, Edgington 1995, 2009, von Fintel 2012, 2013, Gillies 2012, Kaufmann & Kaufmann 2015 — make different choices and thus emphasize different parts of the terrain. There are, in addition, more substantial investigations of the logic of conditionals, including Nute 1980 and (especially) Part II of Veltman 1985. A healthy preoccupation with *if*s should be fed a diet rich in just this kind of variety.

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